Carrier Frequency Offset Estimation for OFDM-Based Cooperative Communications Systems

Mercedes Puzio[†] Gustavo González Fernando Gregorio Juan E. Cousseau[‡]

IIIE-CONICET, Department of Electrical and Computer Engineering – Universidad Nacional del Sur (UNS)
Av. Alem 1253 (B8000CPB) Bahía Blanca, Argentina
† *mercedes.puzio@uns.edu.ar* ‡ *jcousseau@uns.edu.ar*

***Abstract*— We analyze in this work the carrier frequency offset (CFO) estimation issue of OFDM-based cooperative communications systems. We focus on the relaying phase of this scenario, where one or more relaying nodes decode and forward data, originated by a source node, to a single destination node. For an OFDM symbol to be decoded correctly, all its subcarriers must be orthogonal. If any relay node introduce a frequency offset in the assigned subcarriers subset, orthogonality is lost in the whole symbol and data cannot be recovered. A recently proposed training sequence, which consits of one OFDM block with a tile structure and a number of nulled subcarriers, is studied here. By using this subcarrier arrangement, CFO can be estimated with conventional subspace-based methods, such as ESPRIT and MUSIC. In particular, the later achieves a better performance over a wider range of SNR. Despite their low computational complexity, these techniques evidence a high sensitivity to eigenvalue spread, which can be the subject of a future work.**

***Keywords*— Cooperative Systems, Frequency Estimation, CFO, Subspace-Based Methods.**

1 INTRODUCTION

OFDM (Orthogonal Frequency-Division Multiplexing) is a signalling technique specially suitable for frequency-selective fading channels, where the signal bandwidth is much larger than the coherence bandwidth.

OFDM is known for being robust to timing errors, but its main drawback is its sensitivity to CFO. High data rates and small subcarrier (S/C) spacing require CFO to be estimated and compensated to avoid system performance degradation. This is specially critical in the uplink of multi-user communications schemes, such as in OFDMA (Orthogonal Frequency-Division Multiple Access) and OFDM-based cooperative systems.

When different mobile relay nodes forward data (previously broadcasted from a source node) to a destination node over OFDM symbols, we have an OFDM-based cooperative system. Each relay node will introduce a CFO in the assigned subcarriers, which causes the loss of orthogonality among subcarriers in the whole OFDM symbol. Therefore, data becomes unrecoverable.

The motivation of this work is to approach the problem of CFO estimation by considering a recently introduced subspace-based technique, EGC-FBS-ESPRIT (as for *Equal Gain Combination–Forward-Backward Smoothing*–ESPRIT) [1]. From this starting point, the main goal is to build the simulation scenario, compare the obtained results with the widely known MUSIC and continue to explore for improvements of the CFO issue on the line of subspace-based methods for spectral estimation.

Section 2 describes the general OFDM-model, the basics of a cooperative communications context and the signal model we adopt. Section 3 focuses on the recently proposed CFO estimation algorithm. Section 4 analyzes CFO estimation conditions and trade-offs, and presents simulation results in an OFDM-based cooperative scheme using the proposed technique, as well as other subspace-based method. Section 5 elaborates the final thoughts for this work and points to a possible path to follow in the future regarding CFO estimation in cooperative systems.

2 SYSTEM MODEL

2.1 OFDM

In a single-user OFDM model, a high-rate encoded serial data stream at the transmitter input is divided into N parallel substreams. Then, as shown in Fig. 1, these substreams are modulated onto N orthogonal carriers or *subcarriers* through an N-point inverse discrete Fourier transform (IDFT). This increases the symbol duration T_S by a factor of N, adding robustness against intersymbol interference (ISI) caused by frequency-selective fading.

Frequency-selective fading is also associated with time dispersion, where contiguous blocks may partially overlap in time-domain, producing interblock interfer-

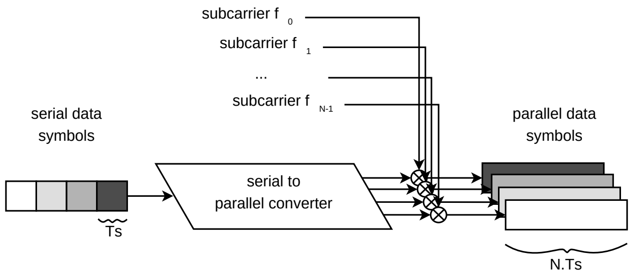

Figure 1: OFDM subcarrier modulation.

ence (IBI). This issue can be mitigated by introducing guard intervals among adjacent blocks, usually a *cyclic prefix* (CP) and, as we introduce in this particular implementation, a *cyclic suffix* (PP). The CP is a sequence of length N_{cp} appended at the beggining of the time-domain block, i.e. the IDFT output, while the PP, of length N_{pp}, is appended at the end. Both CP and PP correspond to the last N_{cp} and the first N_{pp} samples, respectively, of each IDFT output. The extended block of $N_T = N_{cp} + N + N_{pp}$ samples at the transmitter output, where N is the OFDM symbol length (in sampling intervals), is shown in Fig. 2. Immunity to IBI is achieved as long as N_{cp} and N_{pp} are designed according to the channel delay spread, i.e. larger than the channel impulse response (CIR) length L.

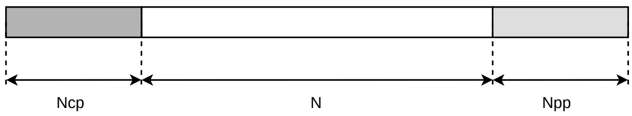

Figure 2: OFDM block, time-domain (IDFT output).

Figure 3 shows a block diagram of a typical OFDM system.

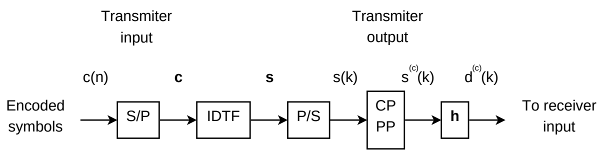

Figure 3: OFDM system block diagram.

Given a data block of length N at the transmitter input $\mathbf{c} = [c(0), c(1), \ldots, c(N-1)]^T$, where $c(n)$ are symbols taken from a particular constellation (e.g. PSK, QPSK or QAM), the IDFT output is:

$$\mathbf{s} = \mathbf{F}^H \mathbf{c} \tag{1}$$

$\mathbf{s} = [s(0), s(1), \ldots, s(N-1)]^T$ is the time-domain signal vector, and $\mathbf{F}$ is the N-point discrete Fourier transform (DFT) matrix with entries given by:

$$[\mathbf{F}]_{n,k} = \frac{1}{\sqrt{N}} e^{-j\frac{2\pi}{N}nk} \tag{2}$$

where $n = 0, 1, \ldots, N-1$ is the frequency-domain (subcarrier) index, and $k = 0, 1, \ldots, N-1$ is the time-domain index.

After vector $\mathbf{s}$ is converted from parallel to serial, the last N_{cp} samples are appended at the beggining (as CP), while the first N_{pp} samples are appended at the end (as PP) of it. This results in the transmitter output vector $\mathbf{s}^{(c)}$:

$$\mathbf{s}^{(c)} = \mathbf{T}^{(c)} \mathbf{s} \tag{3}$$

where $\mathbf{T}^{(c)}$ matrix is:

$$\mathbf{T}^{(c)} = \begin{bmatrix} \mathbf{P}_{N_{cp} \times N} \\ \mathbf{I}_N \\ \mathbf{P}_{N_{pp} \times N} \end{bmatrix} \tag{4}$$

$\mathbf{I}_N$ represents the $N \times N$ identity matrix, while matrices $\mathbf{P}_{N_{cp} \times N}$ and $\mathbf{P}_{N_{pp} \times N}$ collect the last N_{cp} and N_{pp} rows of $\mathbf{I}_N$, respectively.

We consider a time-invariant frequency-selective channel with discrete-time impulse response given by $\mathbf{h} = [h(0), h(1), \ldots, h(L-1)]^T$. Neglecting the contribution of thermal noise for simplicity and assuming there is no IBI, the i-th data block at the receiver input is:

$$\mathbf{d}^{(c)} = \mathbf{B} \mathbf{s}^{(c)} \tag{5}$$

$\mathbf{B}$ is an $N_T \times N_T$ Toeplitz matrix where CIR is arranged as follows:

$$\mathbf{B} = \begin{bmatrix} h(0) & 0 & \cdots & 0 \\ h(1) & h(0) & \cdots & 0 \\ h(2) & h(1) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ h(L-1) & h(L-2) & \cdots & 0 \\ 0 & h(L-1) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & h(0) \end{bmatrix} \tag{6}$$

Defining the matrix $\mathbf{R}^{(c)} = \left[\mathbf{0}_{N \times N_{cp}} \ \mathbf{I}_N \ \mathbf{0}_{N \times N_{pp}}\right]$, CP and PP can be removed from the received data block by using:

$$\mathbf{d} = \mathbf{R}^{(c)} \mathbf{d}^{(c)} \tag{7}$$

The time-domain scalar form of the received signal can be written as:

$$d(k) = \sum_{l=0}^{L-1} h(l) s(k-l) \tag{8}$$

Synchronization is a fundamental task that has to be considered in any digital communications system. Synchronization errors can be either timing, frequency or both.

Loss of timing synchronization results when the time scales at transmitter and receiver sides are not aligned. OFDM is robust to timing errors, since as long as the timing offset remains in the boundaries of the cyclic prefix/suffix, the orthogonality of the transmission is maintained. This allows a simple equalization at the receiver. Timing offset in OFDM is not a critical issue. However, it must be kept small compared to cyclic prefix/suffix length. Approaches to estimation of timing offset in OFDM system have been presented in the literature [5][4], generally based on the fact that the correlation of the received signal with its delayed version reaches a peak when a repetitive training pattern is located.

Due to Doppler shift and/or oscillator frequency drifts, the received carrier frequency f_c may not be exactly equal to the frequency given by the local oscillator f_{lo}. Their difference $f_d = f_c - f_{lo}$ is referred to as *carrier frequency offset* (CFO). One of the main drawbacks of OFDM systems is their sensitivity to CFO, which cause loss of orthogonality between OFDM subcarriers. This can degrade the system performance significantly if it is not estimated and compensated, specially when subcarrier spacing is small and data rates are high. Several techniques have been proposed to estimate and compensate CFO. They can be classified into *data-aided* and *non-data-aided*. Data-aided techniques are based on pilot symbols or a preamble embedded into the transmitted signal. Non-data-aided methods employ the inherent structure of OFDM symbols, e.g. based on null subcarriers [2][1].

If now we consider complex-valued additive white gaussian noise (AWGN), the complete signal model at the receiver can be expressed as:

$$d(k) = e^{j\frac{2\pi}{N}\epsilon k} \sum_{l=0}^{L-1} h(l)s(k-\tau-l) + w(k) \qquad (9)$$

where $\epsilon = f_d N T_s$ is the CFO normalized to the subcarrier spacing $f_{cs} = \frac{1}{NT_s}$, τ is the timing error (in sampling intervals) and $w(k)$ is AWGN with variance σ_w^2.

This work focuses on the estimation of CFO parameters on a cooperative scenario.

2.2 Cooperative Context

Figure 4 shows a *decode and forward* cooperative system with one source node, one destination node and M relay nodes. Two phases can be distinguished in this context: broadcasting and relaying.

In the broadcasting phase, a *training sequence* (used for synchronization purposes) followed by data blocks is broadcasted from the source node to the relay nodes. In the relaying phase, during the synchronization period, the M relay nodes transmit training sequences to the destination node, where a multiple-parameter estimation is performed.

Given an OFDM symbol, each relay node uses only the assigned set of subcarriers, i.e. a subchannel. This introduce different frequency offsets ϵ_i among subcarriers in different subchannels, where $i = 1, \ldots, M$. As consequence, the OFDM symbol loses orthogonality

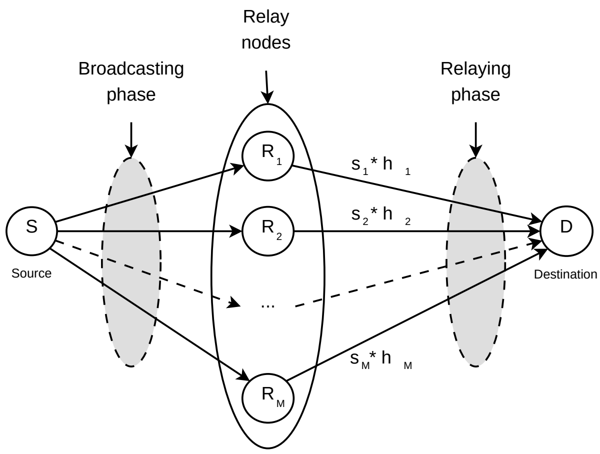

Figure 4: Cooperative system structure.

among its subcarriers and the encoded data cannot be recovered.

CFO compensation cannot be accomplished on the destination node, since the correction of one relay node would misalign the others. Asuming a feedback synchronization scheme (e.g. through a spread-spectrum downlink channel), the destination node feeds the estimated frequency offsets back to the relay nodes, so each one of them can adjust its own frequency parameters. Thus, OFDM symbols arrive frequency-synchronized at the destination node.

Since widely known frequency synchronization techniques for single-user OFDM can be applied in the broadcasting phase, we will focus on CFO estimation in the relaying phase of cooperative systems, by using known training sequences.

2.3 Signal Model

Recalling Eq. (9), the received training signal at the destination node in a cooperative communications system can be expressed as the sum of M single-user OFDM symbols:

$$d(k) = \sum_{i=1}^{M} e^{j\frac{2\pi}{N}\epsilon_i k} \sum_{l=0}^{L-1} h_i(l)s_i(k-\tau_i-l) + w(k) \qquad (10)$$

where $s_i(k)$ is the training sequence at the transmitter output (excluding CP and PP) for $k = 0, \ldots, N-1$, $\mathbf{h}_i = [h_i(0), \ldots, h_i(L-1)]^T$ is the channel impulse response (CIR), τ_i is the normalized timing error (to sampling period) and ϵ_i is the normalized CFO (to subcarrier spacing), which are associated with the i-th relay node, while $w(k)$ is AWGN with variance σ_w^2.

The structure of the training sequence proposed in [1] is shown in Fig. 5.

The useful part of the sequence (i.e. excluding CP and PP) consists on one OFDM symbol of DFT size N (total number of subcarriers). The N subcarriers are divided in P groups of Q subcarriers. A *tile subchannel* is composed by V adjacent subcarriers of each group.

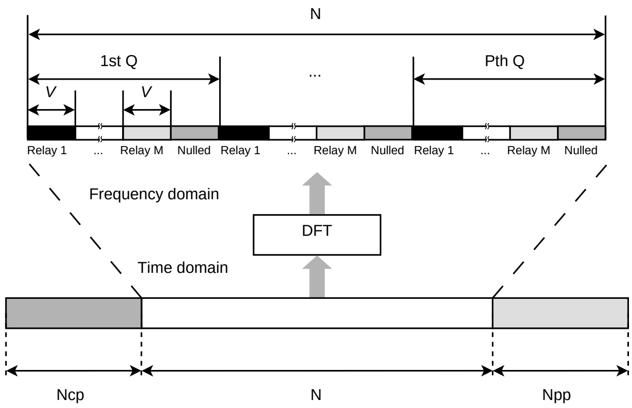

Figure 5: Training structure.

The number of active subcarriers allocated to all relay nodes in each group, that is $M \times V$, is smaller than Q, since null subcarriers are inserted for CFO estimation purposes. Encoded ± 1 symbols used among tiles are random sequences of length V.

Subcarriers with index $\eta_{i,v,p} = v + pQ + \kappa_i P$ compose the tile subchannel assigned to the i-th relay node, where κ_i is an integer in the interval $[0;\ Q/V - 1]$, $v = 0, \ldots, V-1$ and $p = 0, \ldots, P-1$. Thus, the received signal on Eq. (10) can be expressed as:

$$d(k) = \sum_{i=1}^{M} \sum_{v=0}^{V-1} d^{(i,v)}(k) + w(k) \tag{11}$$

where $d^{(i,v)}(k)$ is given by:

$$d^{(i,v)}(k) = e^{j\phi_i} e^{j\theta_{i,v}(k-\tau_i)} \sum_{p=0}^{P-1} S_i(\eta_{i,v,p}) H_i(\eta_{i,v,p}) \cdot e^{-j\frac{2\pi}{P}p(k-\tau_i)} \tag{12}$$

where $\phi_i = 2\pi\epsilon_i\tau_i/N$, $S_i(n)$ and $H_i(n)$ are the frequency responses of the transmitted training sequence and the channel at the n-th subcarriers associated with the i-th relay node, respectively, and:

$$\theta_{i,v} = \frac{2\pi}{N}\left(\epsilon_i + v + \kappa_i V\right) \tag{13}$$

This work considers $|\epsilon_i| < 0.5$, i.e. CFO less than half of the subcarrier separation, which will enable us to apply known subspace-based methods to estimate them, as detailed below.

3 ESTIMATION

From Eq. (11), the collected samples at the receiver can be expressed as:

$$d(l+\mu P) = \sum_{i=1}^{M} \sum_{v=0}^{V-1} e^{j\theta_{i,v}\mu P} d^{(i,v)}(l) + w(l+\mu P) \tag{14}$$

where $l = 0, 1, \ldots, P-1$ and $\mu = 0, 1, \ldots, Q-1$.

If these N samples are arranged into a $Q \times P$ matrix $\mathbf{D} = [\mathbf{D}_0, \mathbf{D}_1, \ldots, \mathbf{D}_{P-1}]$, where the column-vectors are given by $\mathbf{D}_l = [d(l), d(l+P), \ldots, d(l+(Q-1)P)]^T$, we have that:

$$\mathbf{D}_l = \mathbf{G}\mathbf{d}_l + \mathbf{w}_l \tag{15}$$

where $\mathbf{d}_l = \left[d^{(1,0)}(l), \ldots, d^{(i,v)}(l), \ldots, d^{(M,V-1)}(l)\right]^T$, matrix $\mathbf{G} = [\mathbf{G}_{1,0}, \mathbf{G}_{1,1}, \ldots, \mathbf{G}_{i,v}, \ldots, \mathbf{G}_{M,V-1}]$, column-vectors $\mathbf{G}_{i,v} = \left[1, e^{j\theta_{i,v}P}, \cdots, e^{j\theta_{i,v}(Q-1)P}\right]^T$ and $\mathbf{w}_l = [w(l), w(l+P), \ldots, w(l+(Q-1)P)]^T$ as the AWGN column-vector.

The CFO estimation problem requires to find $\theta_{i,v}$ parameters. Recalling Eq. (13), since $|\epsilon_i|$ are assumed to be less than 0.5, $\theta_{i,v}$ are distinct to each other for all (i,v). We notice that Eq. (15) adjusts to the signal model described in [6] for subspace-based spectral estimation methods, if we consider parameters $\theta_{i,v}$ as frequency targets. Since null carriers are present on the training OFDM symbol (i.e. $Q > M \times V$), building the null subspace, then CFO estimation can be performed by using subspace-based methods (such as MUSIC and ESPRIT). These are known to provide low complexity algorithms, if compared to maximum-likelihood (ML) methods.

FBS-ESPRIT (Forward-Backward Smoothing ESPRIT), an ESPRIT-based method, is adapted in [1] to the CFO estimation problem. The proposed steps to estimate $\theta_{i,v}$ are described as follows.

Estimate the covariance matrix of signal vectors $\mathbf{D}_l$, $\mathbf{R}_{dd} = E\left\{\mathbf{D}_l\mathbf{D}_l^H\right\}$, using forward-backward smoothing:

$$\hat{\mathbf{R}}_{dd} = \frac{1}{2}\left(\tilde{\mathbf{R}}_{dd} + \mathbf{J}\tilde{\mathbf{R}}_{dd}^T\mathbf{J}\right) \tag{16}$$

where $\tilde{\mathbf{R}}_{dd} = \frac{1}{P}\mathbf{D}\mathbf{D}^H$ and $\mathbf{J}$ is the $Q \times Q$ exchange matrix:

$$\mathbf{J} = \begin{bmatrix} 0 & 0 & \cdots & 0 & 1 \\ 0 & 0 & \cdots & 1 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 1 & \cdots & 0 & 0 \\ 1 & 0 & \cdots & 0 & 0 \end{bmatrix} \tag{17}$$

Compute the singular value decomposition (SVD) of $\hat{\mathbf{R}}_{dd}$ and arrange its eigenvectors associated with the MV largest eigenvalues $\lambda_1, \lambda_2, \ldots, \lambda_{MV}$ into a $Q \times MV$ matrix $\mathbf{U}_s$.

Arrange the first $Q-1$ rows of $\mathbf{U}_s$ into the matrix $\mathbf{U}_{s1}$ and the last $Q-1$ rows of $\mathbf{U}_s$ into the matrix $\mathbf{U}_{s2}$. Obtain $\{\beta_k\}_{k=0}^{MV-1}$ as the MV eigenvalues of $\mathbf{\Xi}$, where:

$$\mathbf{\Xi} = \left(\mathbf{U}_{s1}^H\mathbf{U}_{s1}\right)^{-1}\left(\mathbf{U}_{s1}^H\mathbf{U}_{s2}\right) \tag{18}$$

Parameters $\theta_{i,v}$ are estimated as:

$$\hat{\theta}_{i,v} = \frac{1}{P}\arg\left(\beta_k\right) \tag{19}$$

where $k = v + (i-1)V$, $v = 0, \ldots, V-1$ and $i = 1, \ldots, M$.

Adopting an equal gain combination (EGC) criteria to compute the CFO estimate for each node, the proposed estimator, referred as EFCE (as for EGC-FBS-ESPRIT-based CFO Estimator), is given by:

$$\hat{\epsilon}_i = \frac{1}{V}\sum_{v=o}^{V-1}\left(\frac{N}{2\pi}\hat{\theta}_{i,v} - v - \kappa_i V\right) \tag{20}$$

where $i = 1, \ldots, M$ and $\kappa_i = i - 1$.

4 ANALYSIS AND SIMULATION

For validation and comparison purposes, this work reproduce the simulation setup as in [1].

We consider a cooperative system with two relay nodes, i.e. $M = 2$, a total of $N = 512$ subcarriers per OFDM symbol and a tile size V ranging from 1 to 5. The training sequence has CP and PP lengths of 64 and 48, respectively. To ensure the condition $Q > M \times V$ required by subspace-based algorithms, the proposed function to compute number of subcarriers per group is:

$$Q = 2^{\lfloor \log_2 MV \rfloor + 1} \tag{21}$$

where $\lfloor a \rfloor$ rounds a to the nearest integer smaller than or equal to a.

The CIR length is $L = 16$ and the channel taps $h_i(l)$ are uncorrelated zero-mean Gaussian random variables with exponential power delay profile $E\left\{|h_i(l)|^1\right\}_{l=0}^{L-1} = Ce^{-0.2l}$, where C is a scalar factor to normalize the total energy of the channel taps to unity. The SNR of each relay node es $\sigma_{ts}^2/N\sigma_v^2$, where σ_{ts}^2 is the training sequence variance. Even though it is not necessary, we asume perfect timing synchronization (i.e. $\tau_i = 0$ for all $i = 1, \ldots, M$).

At this simulation context, not only different realizations of AWGN are considered, but also of the relay nodes channels and the random data at the training structure itself. This conditions are specially suitable for the mobile nature of this scenario.

We noticed experimentally that the estimated covariance matrix $\hat{\mathbf{R}}_{dd}$ results *ill-conditioned* for $MV + 1 < Q < 2MV - 1$, which is equivalent to $50\% < MV/Q < 100\%$. This translates into a great dispersion among the highest and the lowest singular values of $\hat{\mathbf{R}}_{dd}$. Then, if $Q = 2MV - K$ with $K > 1$, then the last $K-1$ $\hat{\theta}_{i,v}$ estimates (with both EFCE and MUSIC algorithms) are mirrored or shifted from the actual value by $-2\pi/P$.

Another important parameter to take into account is P, which is the number of column vectors $\mathbf{D}_l$ (that is $l = 0, 1, \ldots, P-1$) averaged during $\hat{\mathbf{R}}_{dd}$ estimation. Given M, V and Q, we observed that $\hat{\mathbf{R}}_{dd}$ is more likely to be *well-conditioned* for higher values of P.

Since symbol length $N = Q \times P$ and relay nodes M are fixed at this particular setup, two *trade-off* relationships are distinguished: (i) $M \times V$ vs. Q, and (ii) Q vs. P.

For a given V, it is desired to satisfy $MV/Q \leq 50\%$, which is accomplished by increasing Q, but this implies decreasing P. And viceversa. The following table shows this compromise among parameters with the current simulation setup:

M	V	Q	P	N	MV/Q	Null S/C
2	1	4	128	512	50%	256
2	2	8	64	512	50%	256
2	3	8	64	512	75%	128
2	4	16	32	512	50%	256
2	5	16	32	512	62,5%	192

It is clear that the best setup correspond to $V = 1$: smallest MV/Q ratio (higher null-to-data subcarriers ratio) and highest P value (better covariance matrix smoothing). This fact will be confirmed in the simulation plottings below.

Figure 6 shows the CFO estimation performance (in terms of MSE) of EFCE under the conditions proposed in [1], which consist of two relay nodes under noisy, varying channels for tile sizes $V = 1, 2, 3, 4, 5$ and SNR ranging from 0 to 30 dB (in 5 dB–steps).

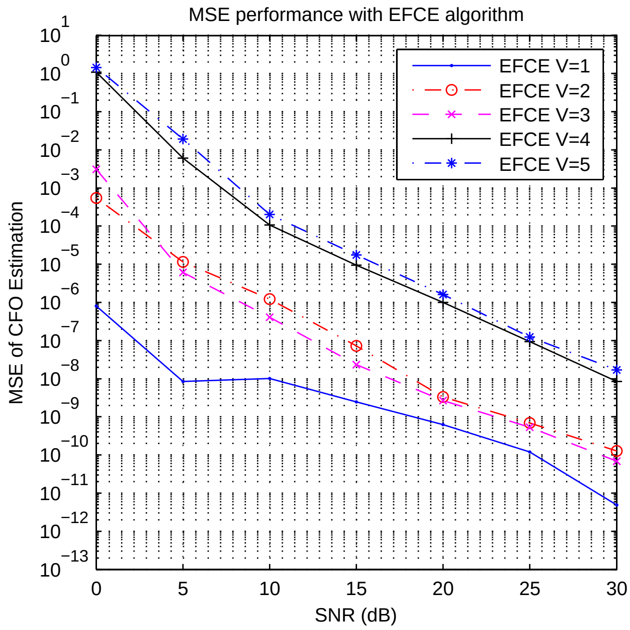

Figure 6: CFO estimation performance of EFCE (MSE averaged over 2 relay nodes, 100 channel realizations and 250 noise realizations).

It can be seen that the best MSE performance is achieved with tile size $V = 1$ in the whole SNR range.

In order to compare simulation results, we recurred to MUSIC, a well known subspace-based estimation method, to estimate $\hat{\theta}_{i,v}$ and then, by adopting an *equal gain combination* criteria, obtain CFO estimates. The covariance matrix R_{dd} computed on Eq. (16) will be also used here.

Figure 7 shows the CFO estimation performance (in terms of MSE) of MUSIC for two relay nodes under noisy, varying channels for tile sizes $V = 1, 2, 3, 4, 5$ and SNR ranging from 0 to 30 dB (in 5 dB–steps).

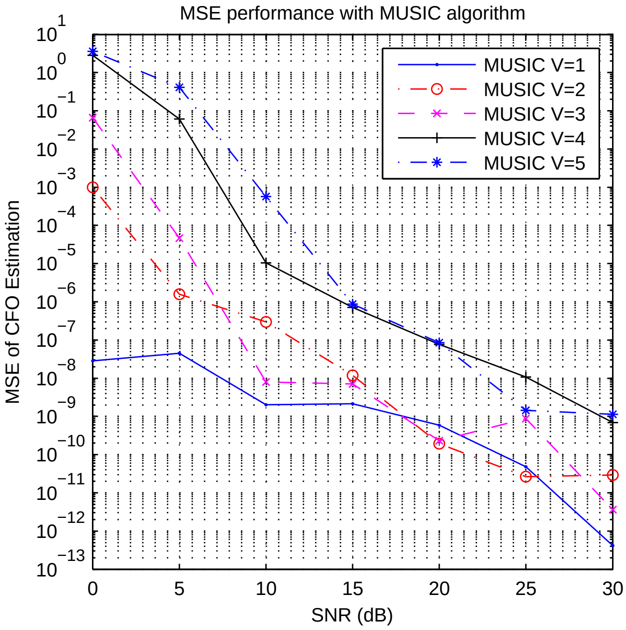

Figure 7: CFO estimation performance of MUSIC (MSE averaged over 2 relay nodes, 100 channel realizations and 250 noise realizations).

CFO estimation performance for EFCE and MUSIC is jointly presented in Fig. 8 for tile sizes $V = 1$ (best performance), $V = 3$ (chosen in [1]) and $V = 5$ (worst performance).

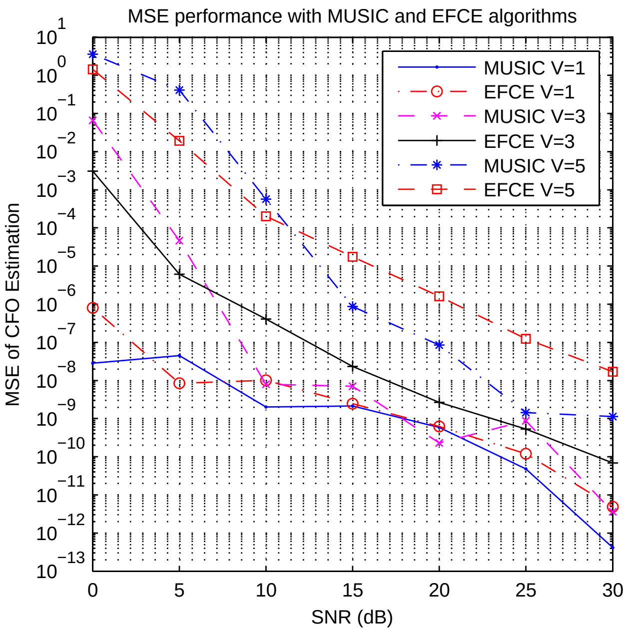

Figure 8: CFO estimation performance comparison between EFCE and MUSIC, $V = 1, 3, 5$ (MSE averaged over 2 relay nodes, 100 channel realizations and 250 noise realizations).

5 CONCLUSIONS

In this work we studied the issue of CFO estimation in a cooperative context and two spectral, subspace-based estimation algorithms were evaluated as a first approach to the problem.

Simulations have shown that EFCE has a better MSE performance than MUSIC for low SNR (e.g. less than 10 dB).

For medium-to-high SNR, the performance results vary on the tile size. Both algorithms show similar performance for small tile sizes, altough MUSIC has a slightly smaller MSE. For large tile sizes such as $V = 5$, MUSIC has clearly a better performance over EFCE.

An important observation is that these subspace-based spectral estimation algorithms have shown high sensitivity to eigenvalue spread of the covariance matrix, specially for low null-to-data subcarriers ratio. This issue may be treated in a future work.

We see in advance a whole line of investigation regarding CFO estimation and subspace-based algorithms that is worth studying, such as [3]. The relatively low complexity of these techniques makes it very attractive for a mobile scenario as cooperative communications, where computational resources and power comsumption are critical issues.

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