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Block-Based Multiple Frequency Estimation Using Complex All-Pass Adaptive Notch Filters

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***Abstract*— This paper introduces a block-based multiple frequency estimation technique for complex sinusoids in the presence of noise. The proposed adaptive method is based on an all-pass notch filter that works sequentially. The algorithm is adjusted to an off-line operation by repeatedly processing the available block of samples. This results in a low complexity alternative to classic subspace methods such as MUSIC and ESPRIT. A typical application field of the new technique is the carrier frequency offset estimation of multiuser multicarrier modulation.**

***Keywords*— Multiple Frequency Estimation, Notch Filters, All-Pass Filters, Lattice Realization, Block Processing.**

1 INTRODUCTION

A common problem in signal processing is the estimation of the frequency of a sinusoidal signal in the presence of broadband (white) noise. To address this issue, several techniques have been developed [1][2].

Applying the maximum likelihood (ML) criterion results in a statistical efficient estimator, but at expense of high computational load. The lower complexity approach of subspace based methods, such as MUSIC, ESPRIT and Pisarenko Harmonic Decomposition (PHD), exploit the covariance matrix structure to estimate the frequency parameters.

An improvement to PHD, the reformed PHD (RPHD), has a finite impulse response (FIR) notch-based approach to estimate the frequency of a real sinusoidal signal in additive white gaussian noise (AWGN) [3]. MUSIC algorithm can also be viewed as an optimum FIR notch design technique, where the solution corresponds to the notch parameters that annihilates the sinusoids component at the input [4].

Notch-based frequency estimation is performed by tuning the parameters of a notch filter to minimize the output variance. Under this condition, notch filter frequencies coincides with the frequencies of the input signal.

Desirable notch filter characteristics such as high noise rejection and sharp cutoff bandpass are achievable with high order FIR structures. The recursive available algorithms for FIR coefficients adaptation require a large amount of iterations to obtain meaningful estimates. Thus, infinite impulse response (IIR) notch filters become an attractive solution [5][6].

A real single-frequency estimator, based on a *direct realization* IIR notch filter, is proposed in [7]. A direct realization notch filter produces biased estimates, which are undesirable under low SNR and complex to correct. On the other hand, the *lattice realization* is not biased, leading to simpler implementations [8][9][10].

Opposite to sequential processing, block-based algorithms work off-line because it is assumed that all the samples of the input signal are available all at once [7][11].

This article presents a block-based technique for multiple frequency estimation of complex sinusoids in AWGN, using an all-pass adaptive notch filter (ANF). The proposed estimation method is based on a sequential ANF. Then, to adapt the filter to an off-line operation, input samples are repeatedly passed through the sequential estimation algorithm. By using this approach, it is shown that an increased estimation performance is obtained, in contrast to passing the input data block once.

The paper is organized as follows. The system model is detailed in Section 2: 2.1 introduces the input signal model, 2.2 proposes a block-based processing for such signal, and 2.3 describes the complex multiple-frequency ANF algorithm. Simulation results are presented and discussed in Section 3. Finally, conclusions are drawn in the last section.

2 SYSTEM MODEL

2.1 Input Signal

The input signal $\tilde{u}(k)$ can be modeled as

$$\tilde{u}(k) = \sum_{i=1}^{M} \alpha_i e^{j(\omega_i k + \phi_i)} + \nu(k), \qquad (1)$$

where $0 \leq k \leq K-1$ is the time index, M is the number of sinusoids, $\nu(k)$ is broadband noise (AWGN), and α_i, ϕ_i and ω_i are the amplitude, initial phase and frequency of the i-th sinusoid, respectively. It is assumed that the amplitude and initial phase are unknown. Our proposal obtains a frequency estimation $\hat{\omega}_i$ of ω_i in (1).

2.2 Block-Based Recursive Approach

The proposed frequency estimation method employs a sequential recursive algorithm A1CANF-S to adapt the coefficients of the ANF. The block-based operation A1CANF-B is achieved by repeatedly passing the samples of the input signal through the algorithm.

Consider $\tilde{\mathbf{u}} = [\ \tilde{u}(0), \cdots, \tilde{u}(K-1)\ ]^T$ as a K-length vector of the samples of (1); and $\mathbf{u}$ as an N-length vector of B concatenated copies of $\tilde{\mathbf{u}}$ (i.e., $N = B \cdot K$),

$$\mathbf{u} = \left[\underbrace{\tilde{\mathbf{u}}^T, \ldots, \tilde{\mathbf{u}}^T, \cdots, \tilde{\mathbf{u}}^T}_{B} \right]^T \tag{2}$$

$$= [\ u(0), \ldots, u(bK), \ldots, u(BK-1)]^T, \tag{3}$$

where $u(k+bK) \equiv \tilde{u}(k)$, $0 \leq k \leq K-1$ is the input signal time index, and $0 \leq b \leq B-1$ is the repetition counter. The input of the block-based algorithm is the vector $\mathbf{u}$ defined in (3). The structure of the ANF is described below.

2.3 Complex All-Pass Based ANF

The multiple frequency, lattice adaptive notch filter adopted in this work was first proposed in [8] and later extended for complex signals in [10]. It is defined as

$$H(z) = \frac{1}{2}\left[1 + \prod_{i=1}^{M} V_i(z)\right]. \tag{4}$$

The term $\prod_{i=1}^{M} V_i(z)$ inside the brackets is a cascade of M all-pass filters $V_i(z)$, tuned to the frequency $\tilde{\omega}_i$,

$$V_i(z) = \frac{\rho_i - e^{j\tilde{\omega}_i} z^{-1}}{1 - \rho_i e^{j\tilde{\omega}_i} z^{-1}}, \tag{5}$$

where ρ_i $(0 < \rho_i < 1)$ is the pole radius of the i-th all-pass section.

To minimize the variance of the output signal $y(n) = H(z)u(n)$, where $n = 0, \ldots, N-1$ is the time index ($N = B \cdot K$), we employ the recursive error prediction method of [5]. This leads to a normalized stochastic gradient algorithm for update notch coefficients $\tilde{\omega}_i$ $(i = 1, \ldots, M)$, defined as

$$\tilde{\omega}_i(n+1) = \tilde{\omega}_i(n) - \frac{\mu}{r_i(n)} \mathrm{Re}\left[y(n)\psi_i^*(n)\right], \tag{6}$$

$$r_i(n+1) = (1-\lambda) r_i(n) + \lambda \left[\psi_i(n)\psi_i^*(n)\right], \tag{7}$$

where $r_i(n)$ is the i-th normalization factor, μ is the step size, and λ is the forgetting factor $(0 < \lambda < 1)$. The stochastic gradient $\psi_i(n)$ is

$$\psi_i(n+1) = -\frac{j e^{j\tilde{\omega}_i(n)} z^{-1}}{D_i(z)} \left[\prod_{k=1, k\neq i}^{M} V_k(z)\right] u(n). \tag{8}$$

The factor $D_i(z)$ corresponds to the denominator of the i-th all-pass section, namely

$$D_i(z) = 1 - \rho_i e^{j\tilde{\omega}_i} z^{-1}. \tag{9}$$

An exponential profile variation for the pole radius ρ_i is considered according to

$$\rho_i(n+1) = \beta_i \rho_i(n) + (1-\beta_i)\rho_i^{\infty}, \tag{10}$$

where β_i defines the exponential decay time constant and ρ_i^{∞} is the asymptotic value of ρ_i.

Algorithm 1 summarizes the pseudocode and Fig. 1 depicts the block diagram of the complex all-pass based multiple notch, where $G_i(z) = -je^{\tilde{\omega}_i(n)} D_i^{-1}(z) z^{-1}$ is the factor outside the brackets of (8).

Algorithm 1 Block-Based Complex Multiple All-Pass ANF (A1CANF-B)

Definitions:
- Input samples $\tilde{u}(k)$ from (1)
- Notch filter transfer function $H(z)$ from (4)
- ANF output signal $y(n)$ $(0 \leq n \leq N-1)$
- All-pass denominator $D_i(z)$ from (9)

Parameters:
- Number of coefficients M
- Number of input samples K
- Number of repetitions B
- Step size μ
- Forgetting factor λ
- Pole radius ρ_i (asymptotic value ρ_i^{∞})
- Exponential decay constant β_i

Initialization:

for $i = 1$ **to** M **do**
  $\tilde{\omega}_i(0) = 0$, $\psi_i(0) = 0$, $r_i(0) = 1$, $\rho_i(0) = 0.9$
end for

Algorithm recursion:

for $b = 0$ **to** $B-1$ **do**
  for $k = 0$ **to** $K-1$ **do**
    for $i = 1$ **to** M **do**
      $n = k + bK$
      $u(n) = \tilde{u}(k)$
      $\psi_i(n+1) = -\frac{je^{j\tilde{\omega}_i(n)} z^{-1}}{D_i(z)} \left[\prod_{k\neq i} V_i(z)\right] u(n)$
      $\tilde{\omega}_i(n+1) = \tilde{\omega}_i(n) - \frac{\mu}{r_i(n)} \mathrm{Re}\left[y(n)\psi_i^*(n)\right]$
      $r_i(n+1) = (1-\lambda) r_i(n) + \lambda |\psi_i(n)|^2$
      $\rho_i(n+1) = \beta_i \rho_i(n) + (1-\beta_i)\rho_i^{\infty}$
    end for
  end for
end for

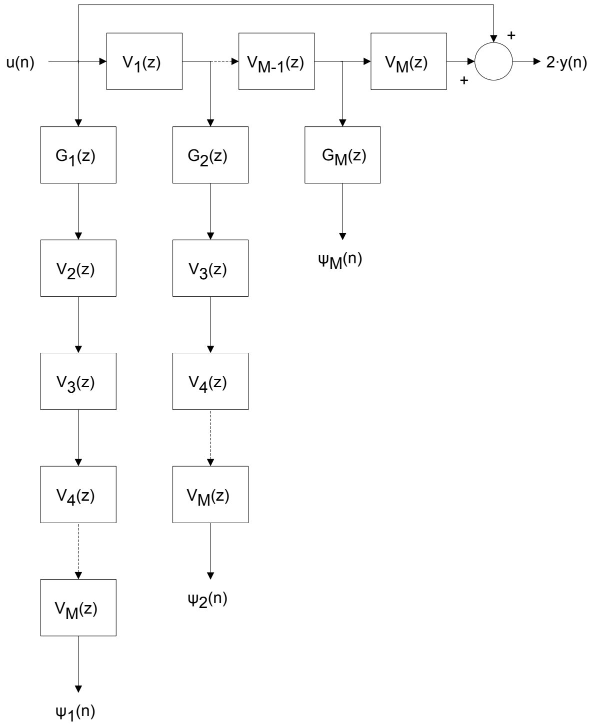

Figure 1: Block diagram of the complex all-pass based multiple notch implementation.

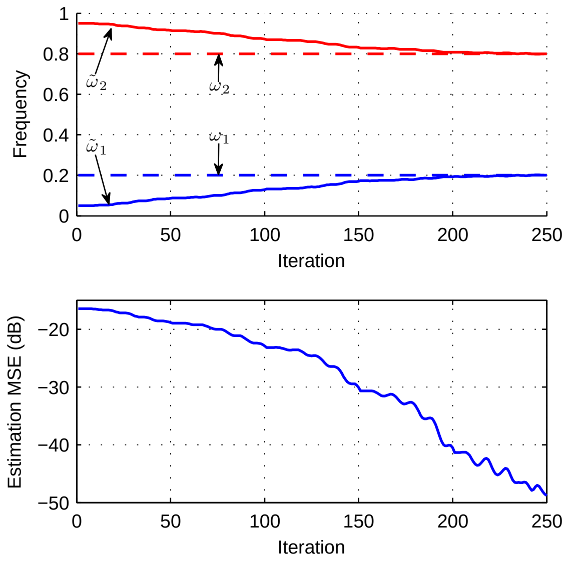

Figure 2: Recursive adaptation of the filter coefficients $\tilde{\omega}_1$ and $\tilde{\omega}_2$ using concatenated repetitions of a single block (A1CANF-B).

3 SIMULATIONS AND DISCUSSION

The performance of the algorithm A1CANF-B is evaluated for two complex sinusoids, with normalized frequencies $\omega_1 = 0.2$ and $\omega_2 = 0.8$, in AWGN. The amplitude of the sinusoids is $\alpha_i = 1$ and the phase ϕ_i is randomly distributed in the interval $[\pi/2, \pi/2)$ for all i. The block length is $K = 50$ samples and $B = 5$ is the number of block repetitions.

Fixed parameters are the step-size $\mu = 8.75 \times 10^{-5}$, the forgetting factor $\lambda = 0.1$, the asymptotic pole radius $\rho_1^\infty = \rho_2^\infty = 0.975$ and the exponential time decay $\beta_1 = \beta_2 = 0.60$. The initial values $\rho_1(0) = \rho_2(0) = 0.750$, $\tilde{\omega}_1(0) = 0.05$, and $\tilde{\omega}_2(0) = 0.95$ correspond to the pole radius and the notch normalized frequencies, respectively.

In Fig. 2 is shown the evolution of notch coefficients $\tilde{\omega}_1$ and $\tilde{\omega}_2$, and their averaged mean-squared error (MSE), as a function of the number of iterations for a signal-to-noise ratio (SNR) of 10 dB. It is observed that the adaptation of the notch coefficients continues through the successive repetitions of K-length blocks. This means that the estimation algorithm based on the stochastic gradient is not able to recover all the information from the first block, and needs the repetition of the signal to converge.

Figure 3 shows the MSE performance versus SNR for tree different approaches: MUSIC, A1CANF-S and A1CANF-B. MUSIC and A1CANF-S use a single input data block, while A1CANF-B uses B repetitions.

A significant performance improvement is achieved by repeating a single data block through the ANF algorithm, i.e. A1CANF-B outperforms A1CANF-S. MUSIC has the best performance for a single block, but at expense of higher complexity. It requires $\mathcal{O}(m^3 + m^2)$ operations, where $M < m < N - 1$ (the choice here is $m = N - 1$). On the other hand, A1CANF-B employs about $\mathcal{O}(10 \cdot MN)$ operations, so it can be seen as a good tradeoff between statistical performance and computational load.

Many problems of multiple frequency estimation with tight constrains in the available number of samples are of interest. A typical example is the carrier frequency offset (CFO) estimation of multicarrier, multi-user/cooperative modulation schemes [12][13][14]. Then, the topic of this paper is of great significance.

4 CONCLUSIONS

A novel block-based estimation technique for multiple complex sinusoids in white noise was proposed. The algorithm is based on all-pass notch filters and employs an stochastic gradient method to perform the estimation. We show that the convergence is significantly improved if the filter input is a concatenation of repetead versions of the available block.

The obtained results encourange the derivation of a block-based adaptation law, with better convergence and performance features.

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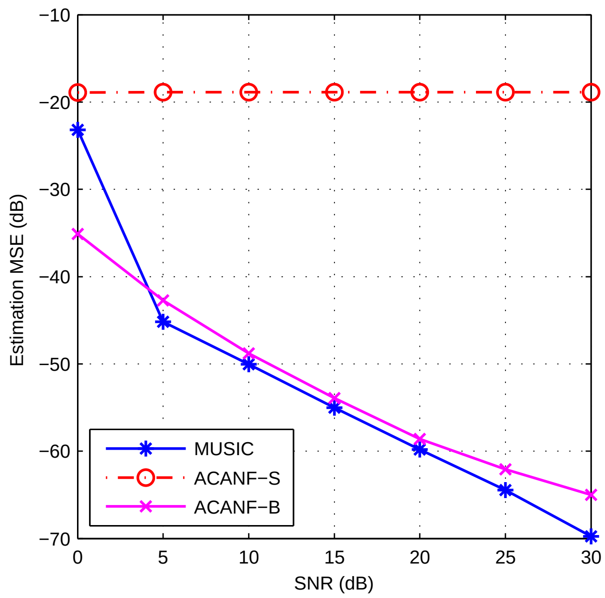

Figure 3: Frequency estimation MSE versus SNR comparison between MUSIC, A1CANF-S and A1CANF-B.

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