Channel prediction for link adaptation in fast fading environments

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Abstract— **We present a channel predictor based on a basis function approach formulation of a Kalman filter channel estimator. The estimator models channel variations via truncated discrete cosine transform (DCT), which is robust to the shape of the channel Doppler spectrum. We show that the DCT can be approximated by a set of linear narrow passband filters that can be formulated as Kalman filters that track a set of decoupled parameters. Because the variation of these parameters is much slower than the channel fading, and the modeling accuracy provided by the filter bank is much better than with traditional models, good prediction performance is achieved by extrapolation in time. Because Doppler frequencies of interest are much lower than the signal bandwidth, a broader prediction horizon is obtained by decimating the estimated channel samples and scaling up in frequency the filter bank accordingly. The derived predictor attains less than 10% power prediction error with a prediction horizon of 1 Doppler wavelength (more than one data frame for typical prediction horizons in cellular systems).**

Keywords— **Fast fading, Channel Estimation, Channel Prediction, Doppler Spread, Kalman.**

I INTRODUCTION

The growing interest on mobile wireless communication systems has opened many interesting research topics on efficient sharing of channel resources between multiple users. Considering time-selective channels, prediction of the mobile radio channel is of particular interest for link adaptation in multiple access systems. For example, in long-term evolution (LTE) [3GPP (2006)] of WCDMA downlink, physical layer scheduler allocates channel resources between users in 2 ms resolution. The resource allocation is based on SNR values reported by the users through a feedback channel. The feedback channel is subject to latency and therefore efficient link adaptation requires prediction of received SNR around 2 ms ahead. This paper presents a novel solution for channel estimation and power prediction for this kind of scenario.

It is known [Rappaport (1996)] that the maximum variation of the wireless channel is upper bounded by the maximum normalized one-sided Doppler bandwidth

$$\nu_{Dmax} = \frac{v_{max} f_C}{c_0} T_S = B_D T_S \tag{1}$$

where f_C is the carrier frequency, v_{max} is the maximum velocity of a user or scatterers, B_D is (unnormalized) Doppler bandwidth, T_S is the symbol duration, and c_0 is the speed of light. In cellular systems, ν_{Dmax} is a small fraction of the signal bandwidth.

Under the assumption of flat fading, the time evolution of the channel is fully described by a sequence of complex scalars at the symbol rate $R_S = 1/T_S$, which is bandlimited to ν_{Dmax}. One typical example of this scenario is OFDM, where a frequency selective channel is transformed into a set of frequency flat subchannels that are easier to estimate and equalize separately than the overall multipath channel.

Channel predictors have been developed exploiting the fact that the signal bandwidth is much larger that the maximum Doppler shift [T. Ekman and Ahlen (2002); M. Sternad and Ahlen (2001); Sternad and Aronsson (2003)]. These approaches are based on FIR predictors and Kalman filter predictors with ARMA models, which approximate Jakes Doppler spectrum model of the time variation of the channel coefficients. The resulting prediction horizons are limited to a fraction of a Doppler wavelength due to mismatch between Jakes Doppler model and practical channel Doppler shapes [X. Zhao and Vainikainen (2003)]. Our approach for channel modeling is based on a deterministic model of the channel variation as in [J. Schmidt and Gregorio (2007); Tsatsanis and Giannakis (1996); Niedzwiecki and Kaczmarek (2005); Zemen and Mecklenbrauker (2005)] which is robust to the shape of Doppler spectrum.

We describe the channel time evolution using a particular orthogonal basis, the discrete time cosine transform (DCT), which can be formulated in terms of a set of narrow passband linear filters. These narrowband filters can be further expressed as Kalman filters that track the coefficients of the basis expansion [J. Schmidt and Gregorio (2007)]. With this Kalman filter formulation, prediction via extrapolation in time is possible. In addition, since the estimator is robust to the shape of Doppler spectrum, highly accurate channel estimates are available facilitating a significantly larger prediction horizon.

The outline of the paper is as follows. In Section II we define the signal model used in the study. In Section III the modeling for the Doppler spectrum is explained, and a low complexity Kalman filter channel estimator is introduced. Section IV describes the derivation of the power predictor and achievable prediction horizons exploiting highly oversampled channel estimates. The extension of the estimator and predictor to frequency selective channels is outlined in Section V. Simulation results are presented in Section VI. Finally, Section VII provides our conclusions.

II SIGNAL MODEL

In this section, we describe the basic signal model for the system under consideration. QPSK symbols are grouped in blocks of length N, where N is the number of subcarriers of an OFDM system. The resulting blocks are processed by an IFFT and preceded by a cyclic prefix to make the convolution of the OFDM symbol with the channel cyclic. The transmission is frame based, where a data frame consists of $M-J$ data symbols and J training symbols placed at the beginning of the frame. Figure 1 depicts the transmitter schematically.

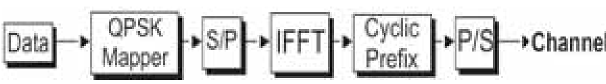

Figure 1: *Model for the OFDM transmitter*

We focus our analysis on the transmission of a symbol sequence $d[m]$ with symbol rate R_S over a time-variant flat-fading channel. The received sequence in the discrete time model is compactly described as the linear system

$$y[m] = h[m]d[m] + z[m], \tag{2}$$

where discrete time is denoted by m, $h[m] \triangleq h(mT_S, 0)$, $d[m]$ is the transmitted symbol sequence, and $z[m]$ is additive circularly symmetric complex white Gaussian noise with zero mean and variance σ_z^2. It is assumed that the channel varies significantly over the duration of a data frame.

III DOPPLER MODELING AND ESTIMATOR DESIGN

For time-varying characterization of the channel, we use a deterministic approach [Zemen and Mecklenbrauker (2005); Niedzwiecki (2000)] which is known to provide better modeling accuracy than the traditional Jakes Doppler model [J. Schmidt and Gregorio (2007)].

If the channel trajectory can be considered to be given by the superposition of R multipath components, each with its particular attenuation and frequency [Tsatsanis and Giannakis (1996)], the time variations of the channel can be modeled by

$$h[m] = \sum_{r=1}^{R} \theta_r e^{j\alpha_r m}, \tag{3}$$

where $\alpha_r = 2\pi f_c \lambda_r$ and θ_r refers to the frequency and the attenuation of multipath component r, respectively.

For characterizing the channel $h[m]$ with a basis expansion we need an efficient representation over the duration of a data frame, that is, for $m \in \{1, \ldots, M\}$. The channel trajectory can be written by means of the inverse DCT (IDCT) (see [Oppenheim and Shafer (1990)]) as:

$$h[m] = \sqrt{\frac{2}{M}} k_u \sum_{u=1}^{M} p[u] \cos\left(\frac{(2m+1)(u-1)\pi}{2M}\right) \tag{4}$$

where $p[u]$ are the DCT coefficients. Since DCT is a linear transform, Eq. 4 allows a representation of the channel without knowing the frequencies nor the attenuations of the multipath components that would require the use of high order statistics [Tsatsanis and Giannakis (1996)].

Theoretical [Jakes (1974); Rappaport (1996)] and experimental results [X. Zhao and Vainikainen (2003)] show that Doppler power spectrum of mobile radio channels is approximately bandlimited to ν_{Dmax}. The energy compaction property of DCT [Oppenheim and Shafer (1990); J. Lee and Chung (1999)] makes it suitable for representing the channel with a small error and a small number of coefficients. Specifically, the Landau-Pollak theorem states that the minimum necessary dimension for the basis expansion is [Lee and Messerschmitt (1994)]:

$$D = \lceil 2\nu_{Dmax} M \rceil + 1 \tag{5}$$

in which, for the Doppler spectrum of interest (Doppler bandwidth much smaller than the signal bandwidth), we have $D \ll M$.

We can then approximate the channel temporal evolution, for $\tilde{h}[m]$, as a linear combination of the cosine basis

$$h[m] \approx \tilde{h}[m] = \sum_{i=1}^{D} f_i[m]\gamma_i = \mathbf{f}^T(m)\mathbf{\Gamma} \tag{6}$$

where $\mathbf{f}(m) = [f_1(m), \ldots, f_D(m)]^T \in \mathcal{R}^{D\times 1}$ is a vector of the basis functions at time instant m, and $\mathbf{\Gamma} = [\gamma_1, \ldots, \gamma_D]^T \in \mathcal{C}^{D\times 1}$ is a vector containing the coefficients of the basis expansion for the actual data frame.

Rewriting Eq. 6, we can express $\tilde{h}[m]$ more conveniently as

$$\tilde{h}[m] = \tilde{h}_1[m] + \tilde{h}_2[m] + \ldots + \tilde{h}_D[m] \tag{7}$$

where we now have

$$\begin{aligned} \tilde{h}_1[m] &= f_1[m]\gamma_1 \\ &\vdots \\ \tilde{h}_D[m] &= f_D[m]\gamma_D \end{aligned} \tag{8}$$

which, since the basis functions are orthogonal, represents a set of decoupled equations to solve for the components of $\mathbf{\Gamma}$. The spectrum of each of these components can be approximated by a narrow passband filter

[J. Schmidt and Gregorio (2007)] in a lattice-form realization [Regalia (1991)] as

$$V_i(z) = \frac{1}{2}(1 - U_i(z)) \tag{9}$$

where

$$U_i(z) = \frac{z^{-2} + \sin\theta_1 i(1+\sin\theta_2)z^{-1} + \sin\theta_2}{1 + \sin\theta_1 i(1+\sin\theta_2)z^{-1} + \sin\theta_2 z^{-2}} \tag{10}$$

for $|\theta_1 i| < \pi/2$, $\sin\theta_1 i = \cos(\omega_0/i)$ and $\sin\theta_2 = \frac{1-\tan(B/2)}{1+\tan(B/2)}$, where $\sin\theta_2$ is a design parameter and B refers to the -3dB bandwidth of the narrowband filter.

With this formulation, each $\tilde{h}_i[m]$ of Eq. 8 can be generated by feeding white complex Gaussian noise e_i to each of the passband filters of Eq. 9. These filters can be expressed in state space form as

$$\begin{aligned} \mathbf{x}_i[m+1] &= \mathbf{F}_i\mathbf{x}_i[m] + \mathbf{G}_i e_i[m] \\ \tilde{h}_i[m] &= \mathbf{H}_i\mathbf{x}_i[m] + B_i e_i[m] \end{aligned} \tag{11}$$

for $i = 1, \ldots, D$. Since the basis functions are symmetric, these matrices are all real valued.

Using Eq. 7 and Eq. 11 the received signal defined in Eq. 2 can be written as

$$y[m] = d[m]\sum_{i=1}^{D}\tilde{h}_i[m] + z[m] \tag{12}$$

The optimum adaptive algorithm for estimating $\tilde{h}[m]$ for white Gaussian noise is a bank of D decoupled Kalman filters [Haykin (1996)]. If we further assume that the transmitted sequence $d[m]$ has constant modulus and is white (QPSK), each Kalman filter admits a steady state solution for the Kalman gain $\mathbf{K}_i[m]$ (for $i = 1, \ldots, D$) [Lindbom (1993); R. Bosisio and Spagnolini (2005)]

$$lim_{t\to\infty}\mathbf{K}_i[m] = \overline{\mathbf{K}} \tag{13}$$

Considering this, low complexity Kalman filters can be derived for estimating the channel [J. Schmidt and Gregorio (2007)]

$$\begin{aligned} \varepsilon[m] &= y[m] - \hat{d}[m]\hat{h}[m] \\ \hat{\mathbf{x}}_i[m+1] &= \mathbf{F}_i\hat{\mathbf{x}}_i[m] + \overline{\mathbf{K}}_i\hat{d}^*[m]\varepsilon[m] \\ \hat{h}[m] &= \sum_{i=1}^{D}\mathbf{H}_i\hat{\mathbf{x}}_i[m] + B_i\varepsilon[m] \end{aligned} \tag{14}$$

The computational complexity of these filters is comparable to that of an LMS algorithm [R. Bosisio and Spagnolini (2005); Lindbom (1993)], while having the tracking performance of a Kalman filter.

IV DERIVATION OF POWER PREDICTOR

Efficient link adaptation requires knowledge of received SNRs in the transmitter. In frequency division duplex (FDD) systems SNR values must be communicated to the transmitter via a feedback channel. Even time-division duplex (TDD) systems require SNR feedback, because transmitter cannot know the level of the interference in the receiver. Realistic feedback channels are subject to feedback latency and therefore channel prediction in the receiver is required, the longer the prediction horizon the better.

One alternative for deriving a channel predictor is to use L-step prediction to extrapolate the estimated complex channel coefficient from Eq. 14 into the future

$$\hat{h}[m+L\,|m] = \sum_{i=1}^{D}\mathbf{H}_i\mathbf{F}_i^L\hat{\mathbf{x}}_i[m\,|m] + B_i\varepsilon[m\,|m] \tag{15}$$

The square of the predicted complex channel tap would then constitute a prediction of the channel power $\hat{p}[m+L\,|m]$ [T. Ekman and Ahlen (2002)]

$$\hat{p}[m+L\,|m] = \left|\hat{h}[m+L\,|m]\right|^2 \tag{16}$$

An appropriate measure for evaluating power prediction algorithms is the normalized mean square power estimation error (NMSE) [Sternad and Aronsson (2003)] defined by

$$NMSE = \frac{E\left|\left|h[m]\right|^2 - \hat{p}[m\,|m-L]\right|^2}{E\left|h[m]\right|^4} \tag{17}$$

As verified experimentally, the achievable prediction horizon with this extrapolation technique is only of 10 to 20 samples with an accurate Doppler modeling as the one presented in Section III, when the power prediction error is below 10%. Compared to one period of maximum Doppler variation (one Doppler wavelength λ), this prediction horizon is generally well below the range required for link adaptation in typical cellular systems. One alternative to broaden the prediction horizon, without increasing the power prediction error, is to use decimated channel samples [T. Ekman and Ahlen (2002)].

In wireless communication systems where channel power prediction is required, the signal bandwidth is usually many times larger than the maximum Doppler frequency component. This characteristic of the time variation of the channel allows to decimate the channel estimates by a factor T without losing information on channel variations. The predictor can then be constructed by scaling up the frequencies of the passband filter bank by the same factor and feeding this new scaled filter bank with the decimated channel estimates [T. Ekman and Ahlen (2002)]. This way, the new prediction horizon is extended to $L' = L \times T$, while keeping the same prediction error, as defined in Eq. 17. Figure 2 shows the estimator/predictor structure schematically.

For the decimation operation on the filter estimates to be feasible, an antialiasing filter must be applied before downsampling. A low complexity solution is to average the channel estimate at the present time instant with previous samples up to 1-5% of the maximum Doppler shift period, where the channel can be considered to remain almost constant.

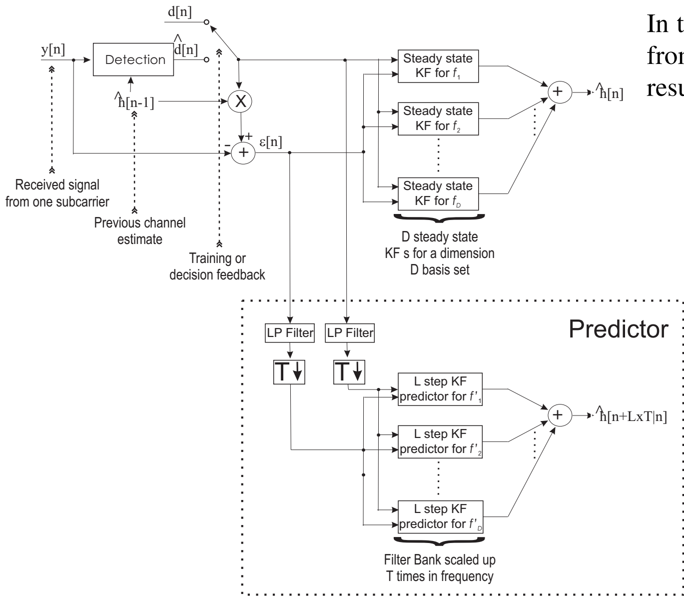

Figure 2: *Estimator/Predictor block diagram*

V EXTENSION TO FREQUENCY-SELECTIVE CHANNELS

For processing at the receiver side, we assume the time-varying channel to remain constant during one OFDM symbol. Therefore, the impulse response of the channel at each time instant m is defined by

$$\mathbf{h}[m] = [h[m,1], h[m,2], \ldots, h[m,P]]^T \in \mathcal{C}^{P\times 1} \quad (18)$$

where P is the number of taps of the impulse response.

The frequency response at time m, $\mathbf{g}[m] \in \mathcal{C}^{N\times 1}$ with elements $g[m,k]$ for $k \in 1,\ldots,N$ is defined as the DFT of the impulse response [Zemen and Mecklenbrauker (2005)]. The receiver removes the cyclic prefix and performs a DFT. The received signal vector after these two operations for each sample time and each subcarrier is given by

$$y[m,k] = g[m,k]d[m,k] + z[m,k] \quad (19)$$

where $z[m,k]$ is complex additive, circularly symmetric white Gaussian noise with zero mean and variance σ_z^2 and $d[m,k]$ is the transmitted symbol at time m on subcarrier k.

According to Theorem 1 of [Negi and Cioffi (1998)], the MMSE estimate of $\mathbf{h}[m]$ can be obtained by using P pilot subcarriers that are equispaced within the OFDM symbol. It is also known [Rappaport (1996)] that the maximum variation of the frequency response across subcarriers, at time instant m, is upper bounded by the coherence bandwidth, which is inversely proportional to the root mean square delay spread of the channel.

Using the results from [Negi and Cioffi (1998)], and Eq. 6 (in frequency domain), a basis set of dimension P can be used to approximate the frequency response of the channel for every instant m where training data is sent. In this way the training overhead can be further reduced from N to only P pilot subcarriers. Figure 3 shows the resulting pilot pattern.

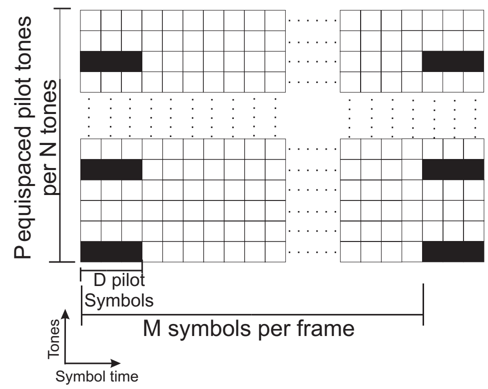

Figure 3: *Pilot structure in OFDM*

VI PERFORMANCE EVALUATION

In this section we evaluate the performance of the proposed predictor. We present simulation results that demonstrate the estimation accuracy obtained by the estimator presented in Section III, from which our power predictor is derived.

We compare our estimator with the estimators derived in [Lindbom (1993)] and [Zemen and Mecklenbrauker (2005)] which we call ARMA2 and DPSS, respectively. These estimators have computational complexity comparable to ours. We evaluate the performance of the three estimators in terms of the MSE for different Doppler models, channel conditions, and training ratios. Two Doppler models are used, one is the classical Jakes Doppler Spectrum [Jakes (1974)] which is widely used in the literature, and the other is a narrow passband spectrum as described in [X. Zhao and Vainikainen (2003)]. Fast fading channels were generated by low-pass filtering complex additive white Gaussian noise to assure that the fading model is realistic.

We consider for simulation the system parameters of [Zemen and Mecklenbrauker (2005)]. The system operates at a carrier frequency $f_C = 2$ GHz with a symbol rate of $R_S = 48.6\times10^3$ Hz. We consider a Doppler bandwidth of $B_D = 160$ Hz which gives $\nu_{Dmax} = 0.0033$, and a maximum Doppler wavelength λ corresponding to 6.23 ms equivalent to 303 symbols. The frame length is set to $N = 256$ symbols. A single subcarrier channel is used for better evaluation of the tracking and prediction capabilities. The dimension of the basis expansion is set to $D = 5$ and the amount of training is set to 1 and 2% (3 and 5 pilots per frame respectively). In all cases we assume the maximum Doppler shift ν_{Dmax} to be known. The only free parameters of our design are: the coefficient $\sin\theta_2$ defined after Eq. 10 (which was set to 0.99), and design parameters L and T.

For the comparison of our channel estimator with ARMA2 and DPSS we first focus on a channel with a

Jakes Doppler spectrum. Figure 4 shows the MSE performance results for this model, which demonstrate the improved modeling accuracy of the basis functions approach compared to the ARMA modeling of Doppler spectrum.

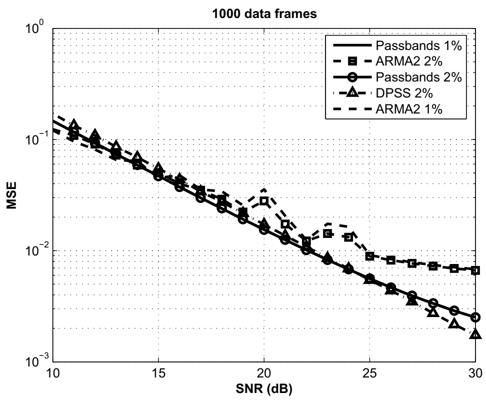

Figure 4: *MSE for* $\nu_{Dmax} = 0.0033$ *and a Jakes Doppler spectrum*

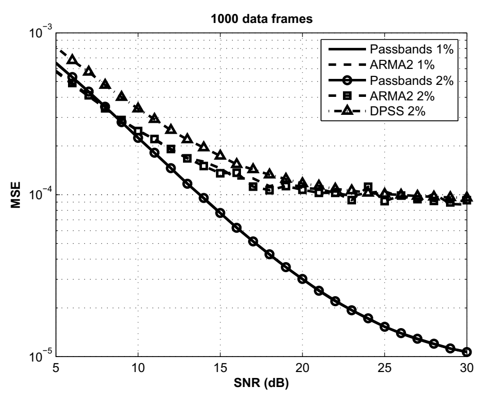

Figure 5: *MSE for* $\nu_{Estimator} = 0.0033$ *and a passband Doppler spectrum centered at* $\nu_{Dmax} = 0.0025$

Figure 5 shows MSE results, this time for a channel with a narrow-passband Doppler spectrum centered at $\nu'_{Dmax} = 0.0025$ (120Hz) that is different from the design parameter $\nu_{Dmax} = 0.0033$ used in the estimators. The results show the main advantage of our estimator when the Doppler spectrum deviates from Jakes model. Notice that ARMA2 estimator is designed to match Jakes Doppler spectrum, and DPSS estimator is designed to best approximate an ideal lowpass spectrum, so it is expected that their performance will degrade when the spectrum is different [X. Zhao and Vainikainen (2003)]. Two independent receiver antennas were used to implement spatial diversity in order to reduce the effects of deep fades [H. Meyr and Fechtel (1998)] in the decision directed mode.

Next we focus on the performance of the predictor in terms of prediction horizon. We first evaluate the power prediction error as a function of the extrapolation factor L with the predictor operating at the symbol rate as in Eq. 15. This allows us to determine the maximum achievable prediction range of our model for the Doppler spectrum. Figure 6 depicts this performance curve together with the channel estimator power error (no Prediction) and the average channel power used as predictor. This figure shows that the passband filter bank attains less than 10% power prediction error for a prediction horizon of $L = 9$. Once the desired power prediction error is set, the value T will determinate the prediction range for this prediction error.

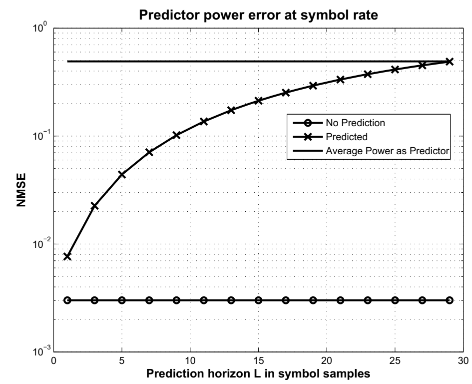

Figure 6: *NMSE for symbol rate prediction and Jakes Doppler spectrum*

Once we know the prediction capability of our estimator, we are able to analyze the choice of the extrapolation factor to be used for extending the prediction range. The maximum allowable decimation factor T to obtain a Nyquist frequency of $2B_D$ is $T = 152$. However, this value of T will require a large order antialias filter for noise reduction. Setting $T = 32$ we obtain 1 Doppler wavelength prediction range for $L = 9$ and a prediction error below 10%. The performance of the predictor for these choices of L and T is shown in Fig. 7 as a function of the signal to noise ratio.

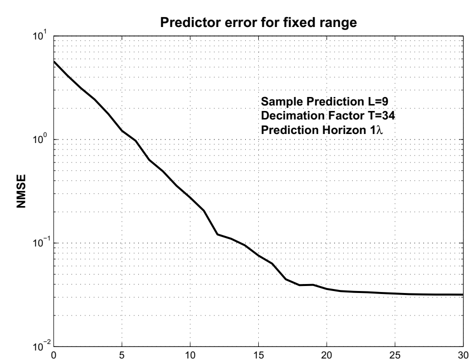

Figure 7: *NMSE vs SNR for 1 Doppler wavelengh prediction range (1λ=303 samples=1.18 frames)*

VII CONCLUSIONS

We developed a channel power predictor based on a low complexity channel estimator for flat fading channels that can be extended to OFDM systems. We showed that a set of narrow passband filters is suitable for the tracking of a fast fading channel if filters' central frequencies are appropriately chosen to approximate an orthonormal basis set. We have included these filters into the formulation of a low complexity Kalman estimator.

The estimator proposed in this work is well suited for channel prediction as it is robust to model changes. Furthermore, the filter structure can be periodically adjusted to match a varying maximum Doppler shift because it is based on very simple basis functions. This results in a channel prediction that attains very little error for prediction horizons as large as one Doppler bandwidth.

The proposed predictor outperforms similar predictor schemes as those reported in [T. Ekman and Ahlen (2002); M. Sternad and Ahlen (2001); Sternad and Aronsson (2003)], mostly due to the better modeling approach for the Doppler spectrum.

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