DIEC

# Introduction - Motivation

- OFDM system:  Discrete model – Spectral efficiency – Characteristics
- OFDM based multiple access schemes
- OFDM sensitivity to synchronization errors

Dto. de Ingeniería Eléctrica y de Computadoras
Universidad Nacional del Sur

DIEC

# OFDM system

Main idea: to divide a high rate data stream into Nu orthogonal carriers, using an N-point IDFT (N > Nu).

Result: high complexity linear equalization required for a frequency selective channel can be replaced by a set of one-tap complex equalizers

Due to channel time dispersion, contiguous blocks may overlap producing IBI. Then, introduction of the cyclic prefix.

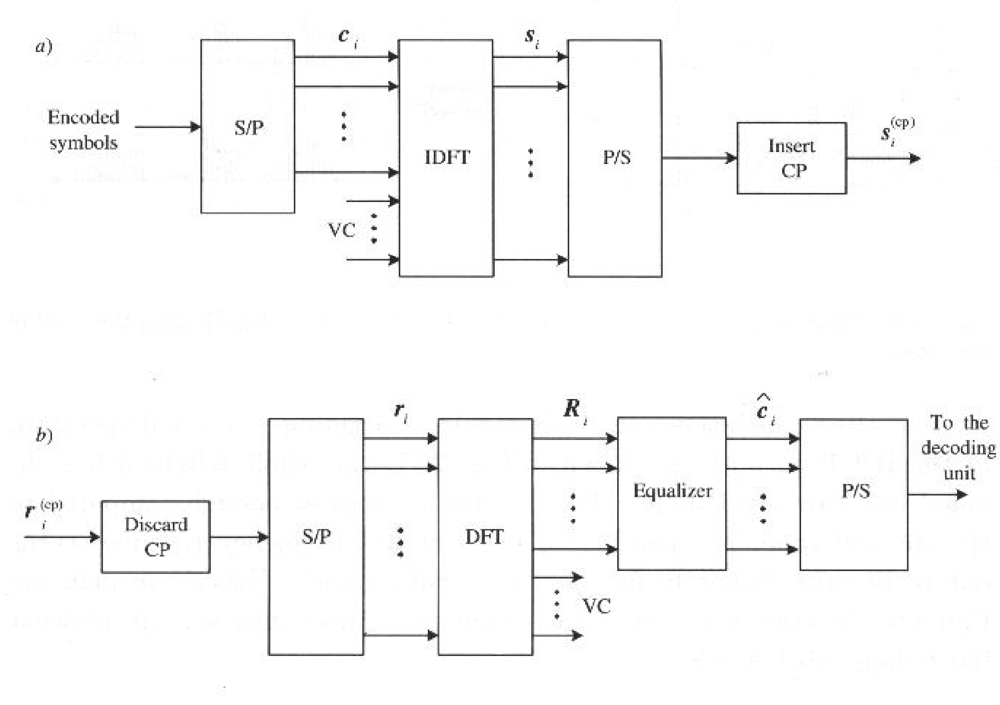



Fig. 2.15 Block diagram of a typical OFDM system: *a*) transmitter; *b*) receiver.

# OFDM discrete-time model (I)

Basic modulation via IDFT

$$\boldsymbol{s}_i = \boldsymbol{F}^H \boldsymbol{c}_i \qquad [\boldsymbol{F}]_{n,k} = \frac{1}{\sqrt{N}} \exp\left(\frac{-j2\pi nk}{N}\right)$$

After P/S converter, addition of the CP (Ng) samples

$$\boldsymbol{s}_i^{(cp)} = \boldsymbol{T}^{(cp)} \boldsymbol{s}_i$$

$$\boldsymbol{T}^{(cp)} = \begin{bmatrix} \boldsymbol{P}_{N_g \times N} \\ \boldsymbol{I}_N \end{bmatrix}$$

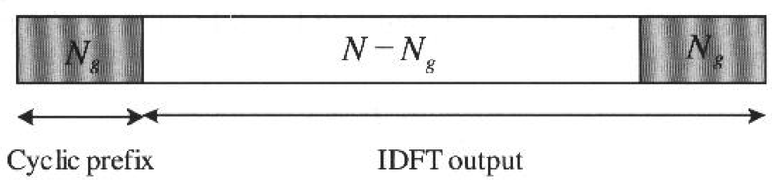



Fig. 2.18 Structure of an OFDM block with CP insertion.

# OFDM discrete-time model (II)

After the channel ***h***, the received block is (not considering noise)

$$\boldsymbol{r}_i^{(cp)} = \boldsymbol{B}^l \boldsymbol{s}_i^{(cp)} + \boldsymbol{B}^u \boldsymbol{s}_{i-1}^{(cp)}$$

$$\boldsymbol{B}^l = \begin{bmatrix} h(0) & 0 & 0 & \cdots & 0 \\ h(1) & h(0) & 0 & \cdots & 0 \\ h(2) & h(1) & h(0) & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ h(L-1) & h(L-2) & h(L-3) & \cdots & 0 \\ 0 & h(L-1) & h(L-2) & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & h(0) \end{bmatrix} \qquad \boldsymbol{B}^u = \begin{bmatrix} 0 & \cdots & 0 & h(1) & h(2) & \cdots & h(L-1) \\ 0 & \cdots & 0 & 0 & h(1) & \cdots & h(L-2) \\ \vdots & \vdots & \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & \cdots & \cdots & \cdots & \cdots & 0 & h(1) \\ 0 & \cdots & \cdots & \cdots & \cdots & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & \cdots & \cdots & \cdots & \cdots & \cdots & 0 \end{bmatrix}$$

## OFDM discrete-time model (III)

If the CP removal is performed by $\boldsymbol{R}^{(cp)} = \left[\ \boldsymbol{0}_{N \times N_g} \quad \boldsymbol{I}_N\ \right]$

$$\boldsymbol{r}_i = \boldsymbol{R}^{(cp)} \boldsymbol{r}_i^{(cp)} = \left( \boldsymbol{R}^{(cp)} \boldsymbol{B}^l \boldsymbol{T}^{(cp)} \right) \boldsymbol{F}^H \boldsymbol{c}_i = \boldsymbol{B}_c \boldsymbol{F}^H \boldsymbol{c}_i$$

where, due to structure of the CP introduced, ***Bc*** is a circulant matrix, that verifies

$$\boldsymbol{F} \boldsymbol{B}_c \boldsymbol{F}^H = \boldsymbol{D}_H$$

where $\boldsymbol{D}_H$ is diagonal with $\boldsymbol{H} = \sqrt{N} \boldsymbol{F} \boldsymbol{h}$ on its main diagonal.

# OFDM discrete-time model (IV)

Hence,

$$\boldsymbol{R_i} = \boldsymbol{D_H c_i}$$

Or, in scalar form

$$R_i(n) = H(n)c_i(n),\ 0 \leq n \leq N-1$$

where

$$H(n) = \sum_{\ell=0}^{L-1} h(\ell) e^{-j2\pi n\ell/N}$$

Is the channel frequency response.

# OFDM discrete-time model (V)

After S/P and DFT, we obtain

$$\boldsymbol{R}_i = \boldsymbol{F}\boldsymbol{B}_c\boldsymbol{F}^H\boldsymbol{c}_i = \boldsymbol{D}_H\boldsymbol{c}_i$$

Finally, data is recovered by

$$\hat{\boldsymbol{c}}_i = \boldsymbol{D}_H^{-1}\boldsymbol{R}_i$$

Since $\boldsymbol{D}_H$ is diagonal, above equation can be written in scalar form as

$$\hat{c}_i = \frac{R_i(n)}{H(n)},\ 0 \leq n \leq N-1$$

corresponding to a bank of one-tap equalizers $1/H(n)$

# OFDM channel equalization basics

Assuming a channel static during the transmission of block I, but that can vary from block to block, the output of the DFT is

$$R_i(n) = H_i(n)c_i + W_i(n),\ 0 \leq n \leq N-1$$

For equalization purposes, in practice, we use $p_i(n)$ and some specific design criterion.

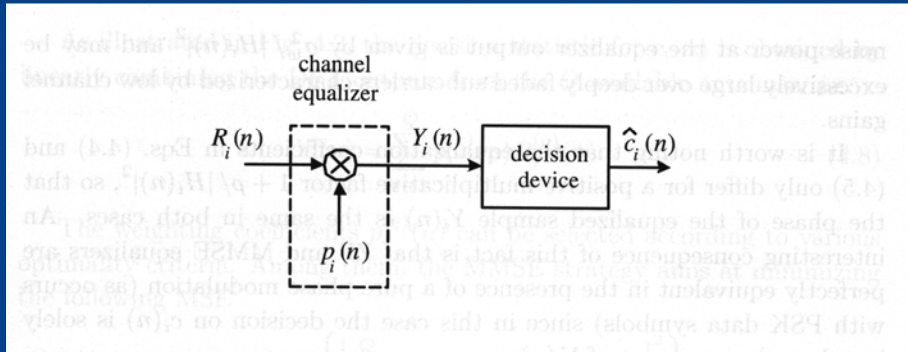


Fig. 4.1 Equalization and data detection over the $n$th subcarrier.

# OFDM channel equalization basics (II)

Minimum mean-square error (MMSE) criterion

$$J_i(n) = E\{|p_i(n)R_i(n) - c_i|^2\}$$

From the orthogonality principle we know that

$$E\{[p_i(n)R_i(n) - c_i]R_i^*(n)\} = 0$$

Then, by computing the expectation with respect to the noise and data (assumed statistically independent with zero mean and variance $C_2$ ) we obtain

$$p_i(n) = \frac{H_i^*(n)}{|H_i(n)|^2 + 1/SNR}$$

# OFDM channel equalization basics (III)

Zero-Forcing (ZF) criterion

Assuming no noise in previous model (or else $SNR \to \infty$ )

$$p_i(n) = \frac{1}{H_i(n)}$$

while the DFT output takes the form

$$Y_i(n) = c_i(n) + \frac{W_i(n)}{H_i(n)}$$

that indicates that ZF equalization compensates any distortion induced by the channel. However, noise power at the equalizer output is increased.

DIEC

# OFDM channel estimation

## Pilot-aided channel estimation

- Transmission of ODFM blocks is usually organized in a frame structure, with some reference known blocks at the beginning to assist the synchronization process. If the length of the frame is shorter than the coherence time, the channel remains constant during the transmission, and initial channel estimation is useful.

- If we consider a time varying scenario, channel change from block to block, and some additional information is required to perform equalization. To this purpose *pilots* are inserted into the payload section of the frame.

DIEC

# OFDM channel estimation (II)

- The pilots are scattered in both time and frequency directions (different blocks and different subcarriers), and are used as reference values for channel estimation and tracking.
- In practice, channel transfer function is first estimated at the positions where pilots are placed, and interpolation techniques are next employed to obtain the channel response over data subcarriers.
- Distribution of pilots in time and frequency depends, logically, on channel characteristics and application.

# OFDM spectral efficiency

If compared with FDM (since overlapping is allowed by orthogonality), higher spectral efficiency is evident in OFDM.

Higher number of subcarriers in an specific bandwidth leads to even better results.

However, also leads to longer blocks that complicate synchronization and/or equalization due to time-selective fading.

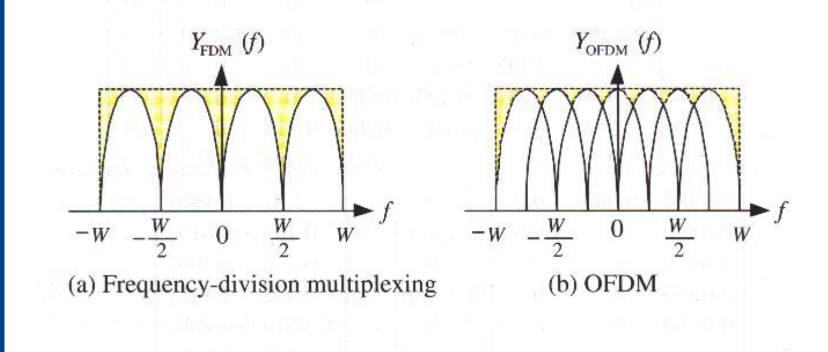



Fig. 2.19 Comparison between the spectral efficiencies of FDM and OFDM systems.

# OFDM characteristics

Main advantages:

- Increased robustness against multipath fading
- High spectral efficiency due to partially overlapping subcarriers.
- Interference suppression capability through the use of cyclic prefix.
- Simple implementation by means of DFT/IDFT.
- Increased protection against narrowband interferences.
- Opportunity to use the subcarriers according to channel conditions.

Drawbacks:

- High sensitivity to frequency synchronization errors (stringent specifications for local oscillators).
- Due to high PAPR, it requires highly linear power amplifiers.
- Loss in spectral efficiency due to the use of the cyclic prefix.

Dto. de Ingeniería Eléctrica y de Computadoras
Universidad Nacional del Sur

DIEC

# OFDM based multiple access schemes

OFDM-TDMA: Not MAI if CP correctly designed. TDMA requires much higher instantaneous power than FDMA.

MC-CDMA: diversity gain by SS, in addition to OFDM advantages. Frequency-selective channel introduces MAI since orthogonality between users is lost (sophisticated multi-user detection is required).

OFDMA: Assuming ideal synchronization, orthogonality is maintained, then MAI is avoided.

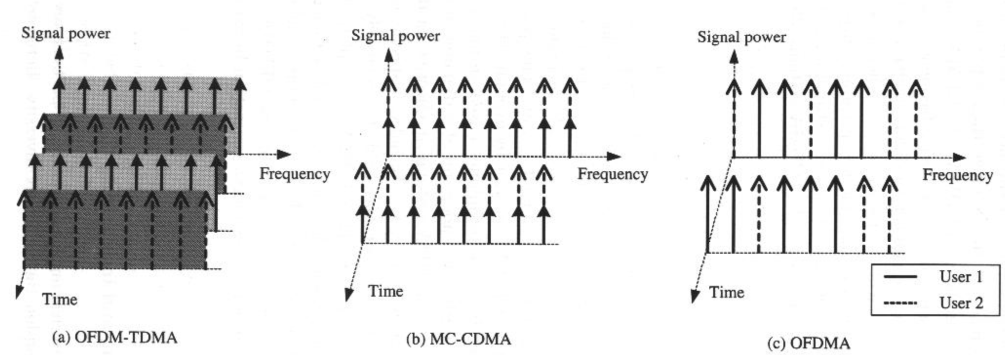



Fig. 2.20 Illustration of OFDM-based multiple-access schemes.

DIEC

# OFDM Sensitivity to synchronization errors

- Sampling clock synchronization, slightly different clocks at transmitter and receiver produces ICI. No practical effects in modern systems.
- Timing synchronization, identification of the beginning of the block, or either the frame, to find the right position of the DFT window.
- Frequency synchronization, differences between carrier and local oscillator for demodulation results in a loss of orthogonality.

DIEC

# OFDM Sensitivity… (II)

The time-domain samples of the i-th OFDM block are

$$s_i^{(cp)}(k) = \frac{1}{\sqrt{N}} \sum_{n \in \mathcal{I}} c_i(n) e^{j2\pi nk/N}, \qquad -N_g \leq k \leq N-1$$

and the equivalent baseband signal transmitted is given by

$$s_T(k) = \sum_i s_i(k - iN_T)$$

# OFDM Sensitivity… (III)

We define the difference between the local oscillator and the carrier frequency, normalized to the subcarrier, as the *normalized carrier frequency offset* (CFO),

$$f_d = f_c - f_{LO}, \qquad \epsilon = f_d/(1/NT_s)$$

and also $\theta$ the number of samples by which the received time scale is shifted from its ideal setting. Then, the received signal is

$$r(k) = e^{j2\pi\epsilon k/N} \sum_i \sum_{l=0}^{L-1} h(l)s_i(k-\theta-l-iN_T) + w(k)$$

Dto. de Ingeniería Eléctrica y de Computadoras
Universidad Nacional del Sur

DIEC

# OFDM Sensitivity (IV)

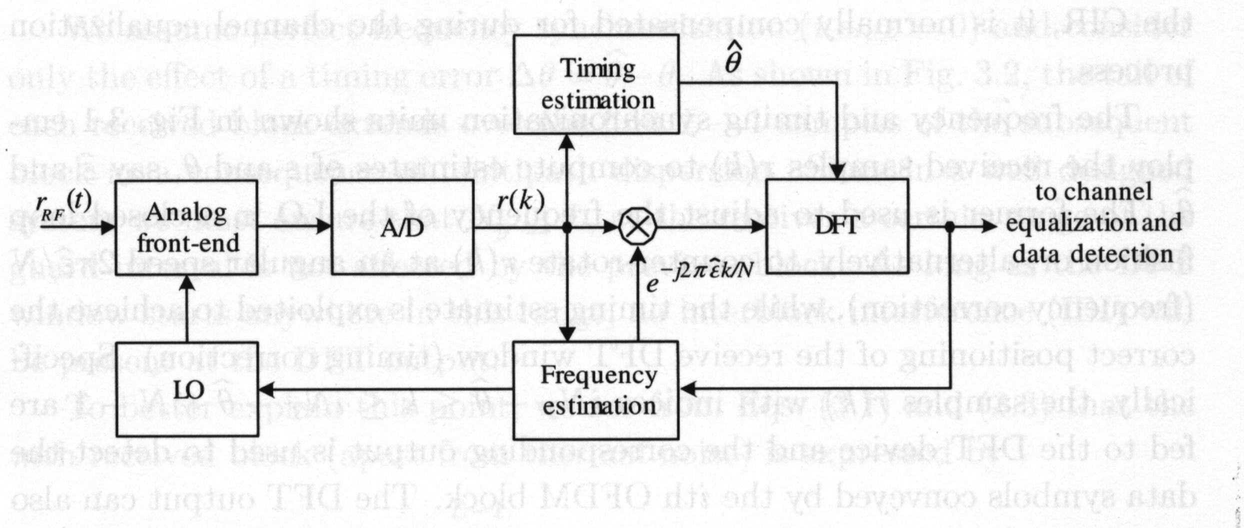


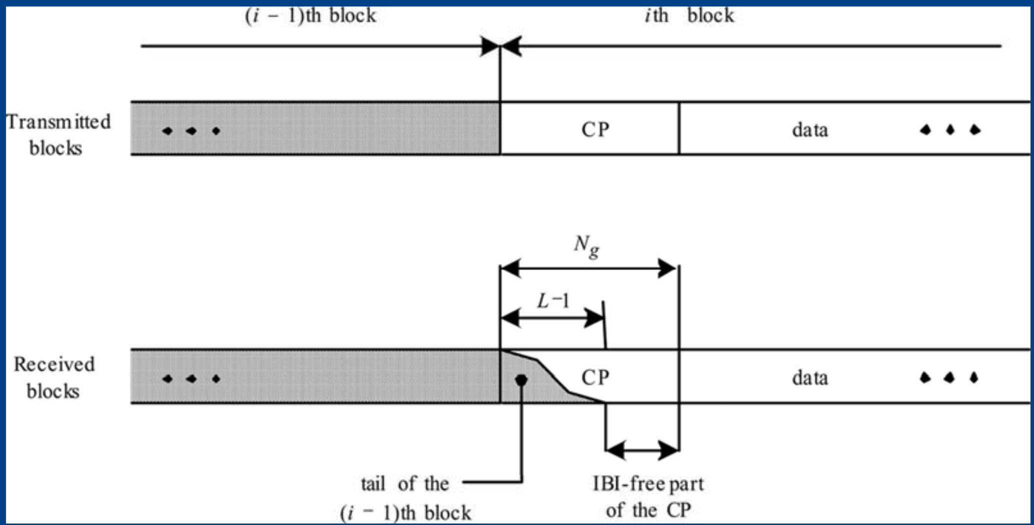

Fig. 3.1 Block diagram of an OFDM receiver.

DIEC

# Timing offset effects (I)

Assuming perfect frequency synchronization and a well designed system, i.e. , $N_g \geq L$ then at the receiver a certain range of the guard interval is not affected by previous block. If the DFT starts anywhere in this range, no IBI will be present.

Defining $\Delta\theta = \hat{\theta} - \theta$
if $-N_g + (L-1) \leq \Delta\theta \leq 0$, no IBI!



# Timing offset effects (II)

The DFT output over the n-th subcarrier can be represented by

$$R_i(n) = e^{j2\pi\Delta\theta/N} H(n) c_i(n) + W_i(n)$$

Since timing offset appears as a linear phase across the DFT outputs, it can be compensated by the channel equalizer.

This means that no single correct timing synchronization point exists in OFDM, since there are $Ng - L + 2$ of them.

# Timing offset effects (III)

If $N_g - (L-1) \leq \Delta\theta$ or $\Delta\theta \geq 0$ there will be not only IBI but also loss of orthogonality among subcarriers, i.e., ICI.

Then, the n-th DFT output can be written as

$$R_i(n) = e^{j2\pi\Delta\theta/N}\alpha(\Delta\theta)H(n)c_i(n) + I_i(n,\Delta\theta) + W_i(n)$$

where $\alpha(\Delta\theta)$ is an attenuation factor and $I_i(n,\Delta\theta)$ reflects IBI and ICI and can be modeled as zero-mean with power $\sigma_I^2(\Delta\theta)$ .

Dto. de Ingeniería Eléctrica y de Computadoras
Universidad Nacional del Sur

DIEC

# Timing offset effects (IV)

An indicator to evaluate the effects of timing error is the *loss in SNR*, as defined by

$$\gamma(\Delta\theta) = \frac{SNR^{ideal}}{SNR^{real}}$$

$$\gamma(\Delta\theta) = \frac{1}{\alpha^2(\Delta\theta)}\left(1 + \frac{\sigma_I^2(\Delta\theta)}{\sigma_w^2}\right)$$

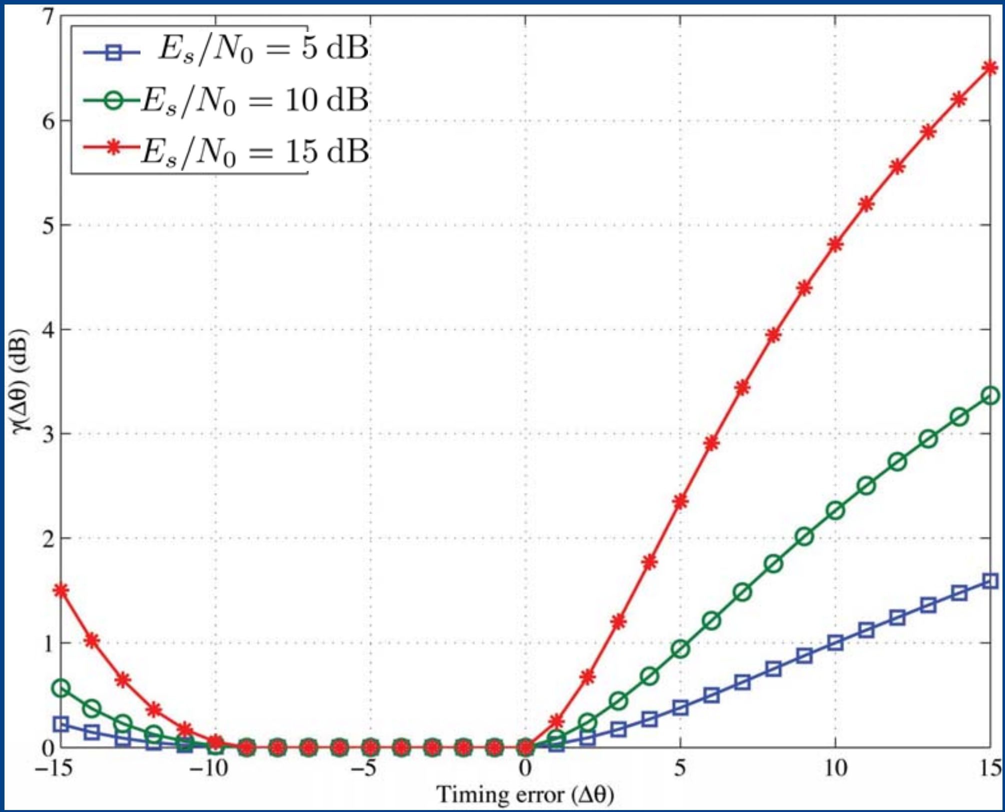
Dto. de Ingeniería Eléctrica y de Computadoras
Universidad Nacional del Sur

DIEC

# Timing offset effects (V)

Example:

- N= 256, Ng=16.
- Rayleigh fading channel with L=8 and exponentially decaying power delay profile.




For a given timing error SNR loss increases with Es/No (at low SNR the main impairment is thermal noise).

To keep SNR loss tolerable (less than 1 dB) residual error after timing correction should be a few percents of N.

DIEC

# Frequency offset effects (I)

Assuming now perfect timing synchronization, at the receiver DFT output the *i* th OFDM block can be written as

$$R_i(n) = e^{j\varphi_i} \sum_{m \in \mathcal{I}} H(m) c_i(m) e^{j\pi(N-1)(\epsilon+m-n)/N} f_N(\epsilon + m - n) + W_i(n)$$

$$f_N(x) = \frac{\sin(\pi x)}{N \sin(\pi x / N)} \qquad \varphi_i = 2\pi i \epsilon N_T / N$$

Dto. de Ingeniería Eléctrica y de Computadoras
Universidad Nacional del Sur

DIEC

# Frequency offset effects (II)

Considering the case in which the frequency offset is a *multiple* of the subcarrier spacing 1/NT, then

$$R_i(n) = e^{j\varphi_i} H(|n-\epsilon|_N) c_i(|n-\epsilon|_N) + W_i(n)$$

then orthogonality is not destroyed and that frequency offset only results into a shift of the subcarrier indices.

Dto. de Ingeniería Eléctrica y de Computadoras
Universidad Nacional del Sur

DIEC

# Frequency offset effects (III)

For the case in which the frequency offset is a fraction of the subcarrier spacing, then

$$R_i(n) = e^{j[\varphi_i + \pi\epsilon(N-1)/N]} H(n) c_i(n) f_N(\epsilon) + I_i(n,\epsilon) + W_i(n)$$

where $I_i(n,\epsilon)$ accounts for ICI.

Letting $E\{|H(n)|^2\} = 1$ as before, and assuming i.i.d. data symbols with zero mean and power Es, the ICI term has zero mean and its power can be written as

$$\sigma_I^2(\epsilon) = E_s(1 - f_N^2(\epsilon))$$

# Frequency offset effects (IV)

Using the SNR loss to study the impact of the frequency errors, the following can be obtained

$$\gamma(\epsilon) = \frac{SNR^{ideal}}{SNR^{real}} \qquad \gamma(\epsilon) = \frac{1}{f_N^2(\epsilon)}\left(1 + \frac{E_s}{N_0}(1 - f_N^2(\epsilon))\right)$$

That, for small values of the frequency offset, can be approximated to

$$\gamma(\epsilon) \cong 1 + \frac{1}{3}\frac{E_s}{N_0}(\pi\epsilon)^2$$

DIEC

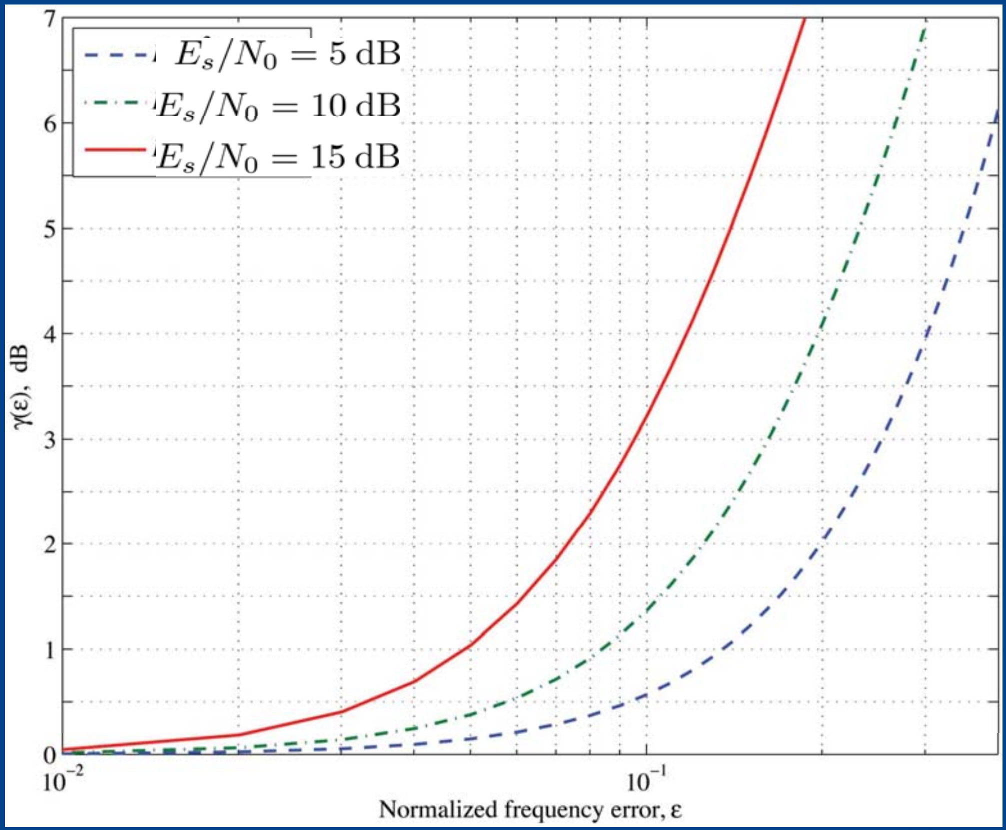
# Frequency offset effects (V)

Example:

Same OFDM and channel parameters than in previous example.

To avoid severe degradation frequency offset should be as low as 4-5 % of subcarrier spacing.

WIMAX, 1/NT = 11.16 kHz, then tolerable freq. offset 500 Hz. For a carrier = 5 GHz, this corresponds to an oscillator instability of 0.1 ppm. For practical designs this requires frequency offset estimation and correction.



Dto. de Ingeniería Eléctrica y de Computadoras
Universidad Nacional del Sur

DIEC

# Conclusions

- OFDM concept introduce considerable simplifications in terms of equalization of frequency selective channels.
- Sensitivity of OFDM to timing and frequency offset errors must be counteracted in any practical implementation.
- Timing errors can be reduced with proper design of the cyclic prefix.
- Only small carrier frequency offset can be tolerated if ICI is expected not to affect BER performance in OFDM.