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Downlink time and CFO synchronization

1. OFDMA transmitter – Downlink synchronization tasks
2. Downlink acquisition (coarse synchronization)
3. Downlink tracking (fine synchronization)

Downlink synchronization tasks (I)

- Similar tasks that performed with conventional single-user OFDM.
- MU uses the broadcast signal transmitted by BS to get timing and frequency estimates, that will be used to control the position of the DFT window and the frequency of the local oscillator.
- Synchronization process is split into an acquisition step followed by a tracking phase.

Downlink synchronization tasks (II)

- *Acquisition process*: pilot blocks with a particular repetitive structure are used to get initial estimates of the synchronization parameters. Specific algorithms must cope with large errors. Phases: frame detection, timing and CFO estimation.

- *Tracking process*: Devoted to maintain and/or refine initial timing and frequency estimates, and also counteract small short-term variations due to oscillator drifts and Doppler shifts.

Downlink synchronization tasks (III)

Simplified frame structure includes:

- Null block (no signal), at the beginning, to estimate interference and noise power.
- Reference blocks, known structure and/or symbols (acquisition tasks).
- Pilot tones: subcarriers with known symbols (tracking tasks).

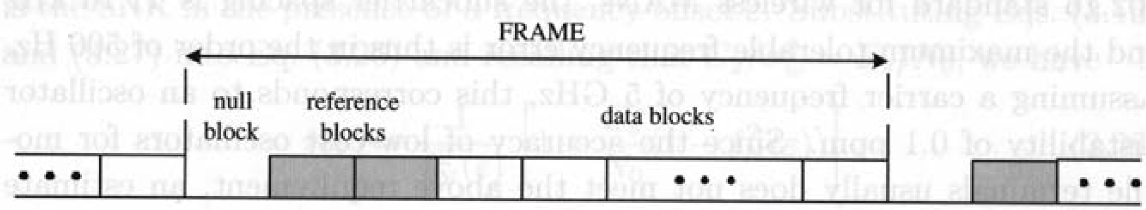
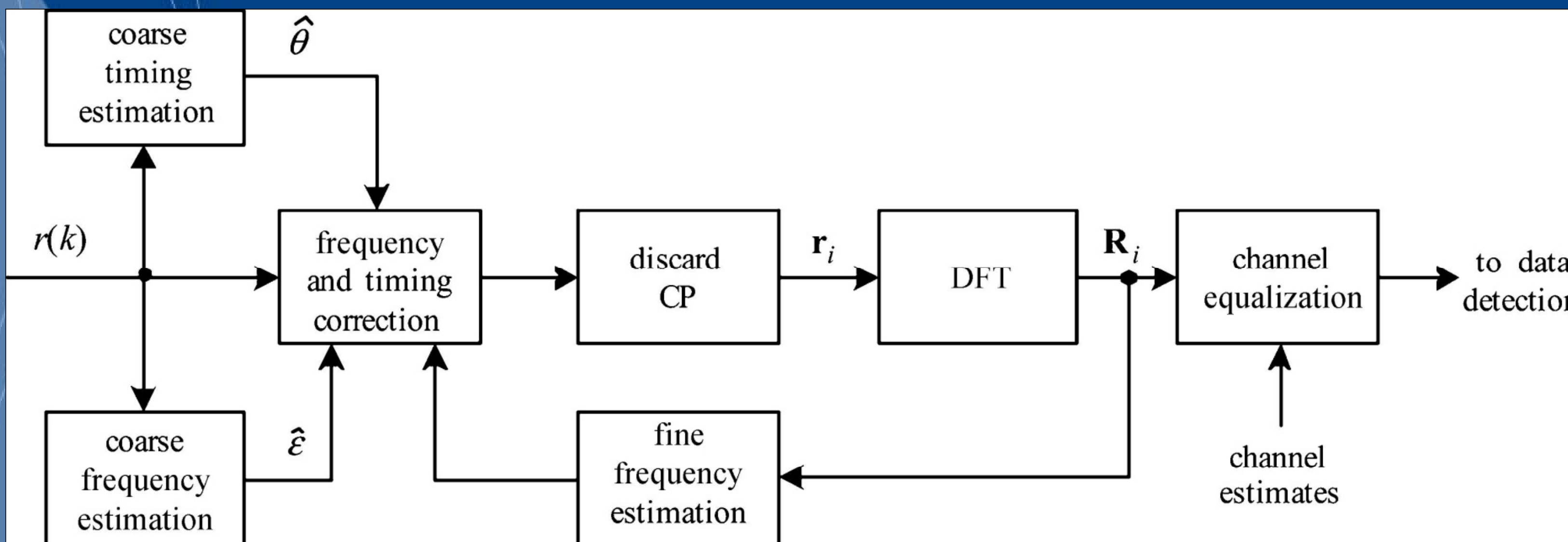


Fig. 3.5 Example of frame structure in the downlink.

Downlink synchronization tasks (IV)



Block diagram of OFDMA downlink receiver

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Downlink timing acquisition

S-C algorithm: Use of reference block with two identical halves (in time domain). This can be generated, in the frequency domain, modulating only subcarriers with even indices with a PN sequence $\boldsymbol{c} = [c(0), c(2), \cdots, c(N-2)]^T$, and setting to 0 the subcarriers with odd indices.

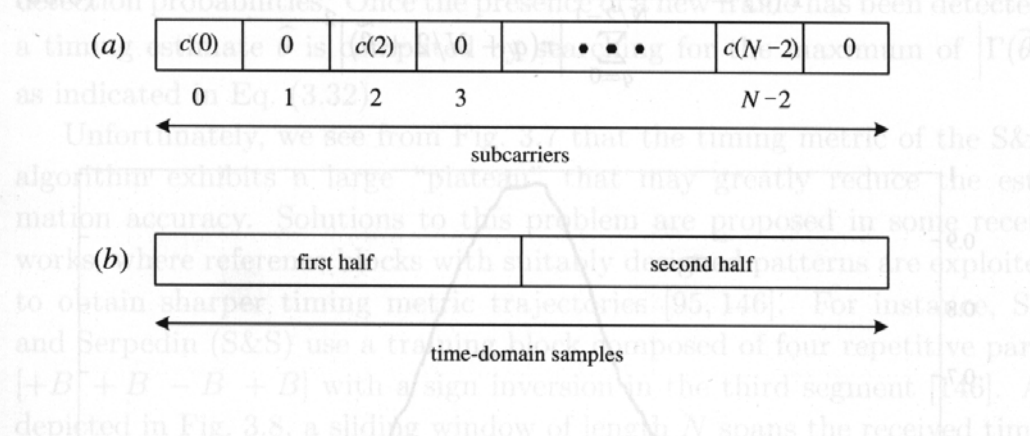


Fig. 3.6 S&C reference block in the frequency-domain (a) and in the time-domain (b)

Downlink timing acquisition (II)

If the CP is properly designed, both halves remain identical after passing through the channel except for a phase difference caused by the CFO. Hence,

$$r(k) = s_R(k)e^{j2\pi\epsilon k/N} + w(k), \qquad \theta \leq k \leq \theta + N/2 - 1$$

$$r(k + N/2) = s_R(k)e^{j2\pi\epsilon k/N}e^{j\pi\epsilon} + w(k + N/2), \qquad \theta \leq k \leq \theta + N/2 - 1$$

Downlink timing acquisition (III)

Then, the magnitude of a sliding window correlation of lag N/2 gives a peak when the window is aligned with the reference block

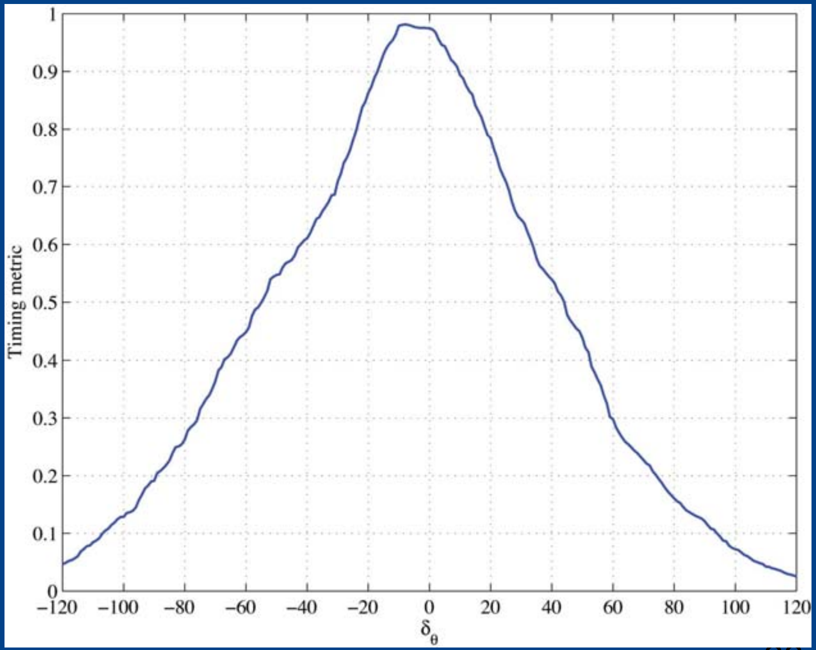
$$\hat{\theta} = \arg\max_{\hat{\theta}}\{|\Gamma(\tilde{\theta})|\} \qquad \Gamma(\hat{\theta}) = \frac{\sum_{q=0}^{N/2-1} r(q+N/2+\tilde{\theta})r^*(q+\tilde{\theta})}{\sum_{q=0}^{N/2-1}\left|r(q+N/2+\tilde{\theta})\right|^2}$$

For frame detection, $|\Gamma(\tilde{\theta})|$ is compared against a given threshold λ , designed to achieve a reasonable trade-off between false alarm and mis-detection probabilities.

Downlink timing acquisition (IV)

Example: Obtained with N=256, Ng=16, Rayleigh multipath channel, L=8 and SNR= 20 dB.

Timing metric of S-C algorithm exhibits considerable ambiguity that reduce the estimation accuracy.



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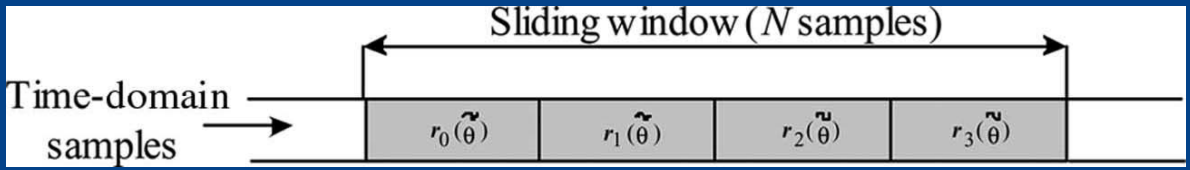
Downlink timing acquisition (V)

S-S algorithm: Use of a training block composed of 4 repetitive parts, defined by: $[+B \;\; +B \;\; -B \;\; +B]$

A sliding window spans the received time-domain samples with indices $\tilde{\theta} \leq k \leq \tilde{\theta} + N - 1$, that defines 4 vectors

$$\boldsymbol{r}_j = \{r(k + jN/4 + \tilde{\theta}); 0 \leq N/4 - 1\}, \qquad \text{with } j = 0, 1, 2, 3$$



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Downlink timing acquisition (VI)

Timing metric of SS algorithm is computed as

$$\Gamma_{SS}(\hat{\theta}) = \frac{|\Lambda_1(\tilde{\theta})| + |\Lambda_2(\tilde{\theta})| + |\Lambda_3(\tilde{\theta})|}{\frac{3}{2}\sum_{j=0}^{3}\left\|\boldsymbol{r}_j(\tilde{\theta})\right\|^2}$$

where

$$\begin{aligned}
\Lambda_1(\tilde{\theta}) &= \boldsymbol{r}_0^H(\tilde{\theta})\boldsymbol{r}_1(\tilde{\theta}) - \boldsymbol{r}_1^H(\tilde{\theta})\boldsymbol{r}_2(\tilde{\theta}) - \boldsymbol{r}_2^H(\tilde{\theta})\boldsymbol{r}_3(\tilde{\theta}) \\
\Lambda_2(\tilde{\theta}) &= \boldsymbol{r}_1^H(\tilde{\theta})\boldsymbol{r}_3(\tilde{\theta}) - \boldsymbol{r}_0^H(\tilde{\theta})\boldsymbol{r}_2(\tilde{\theta}) \\
\Lambda_3(\tilde{\theta}) &= \boldsymbol{r}_0^H(\tilde{\theta})\boldsymbol{r}_3(\tilde{\theta})
\end{aligned}$$

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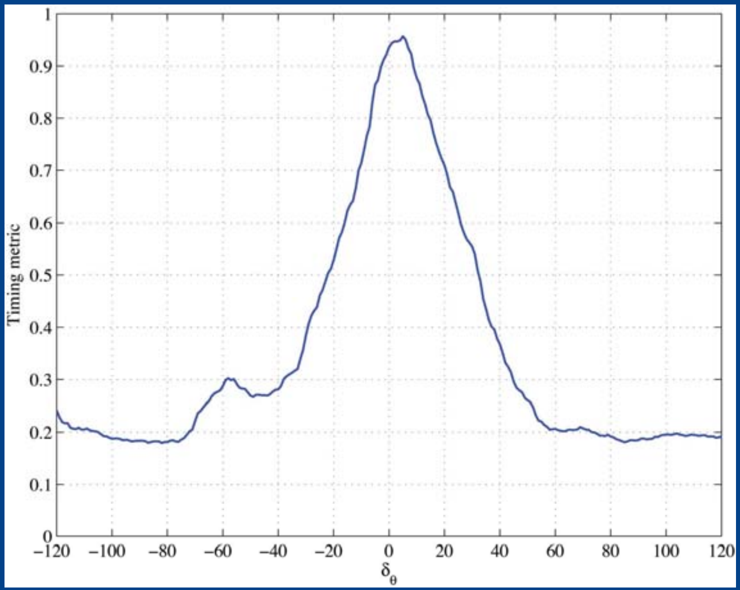
Downlink timing acquisition (VII)

Example: Same as previous example.



Downlink Frequency acquisition

Moose algorithm: assuming time acquisition has been achieved, let $R_1(n)$ and $R_2(n)$ be the *n*th DFT output corresponding to the two reference blocks. Then,

$$R_1(n) = S_R(n) + W_1(n) \qquad R_2(n) = S_R(n)e^{j2\pi\epsilon N_T/N} + W_2(n)$$

An estimate of the CFO is

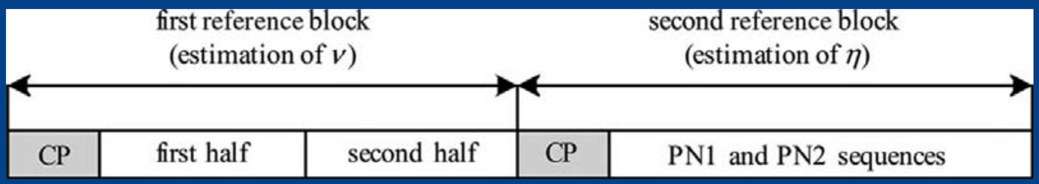
$$\hat{\epsilon} = \frac{1}{2\pi(N_T/N)} \arg\left\{\sum_{n=0}^{N-1} R_2(n)R_1^*(n)\right\}$$

Main drawback: short acquisition range. Since $\arg\{.\}$ return values in $[-\pi, \pi)$, then is must be

$$|\hat{\epsilon}| \leq N/(2N_T)$$

Downlink Frequency acquisition (II)

S-C algorithm: Uses also 2 reference blocks, but with the following structure



First block the same used for timing acquisition. 2nd block contains a differentially encoded PN1 on even subcarriers and another PN2 on odd subcarriers.

Downlink Frequency acquisition (III)

Assuming timing acquisition done, with θ successfully estimated, frequency error is decomposed into a fractional part (less than *1/T* in magnitude) and an integer part (multiple of *2/T*), with *T* = *NTs*.

Then the normalized CFO can be written as

$$\epsilon = \nu + 2\eta \qquad \nu \in (-1, 1], \text{and } \eta \text{ is an integer.}$$

S-C algorithm uses the first reference block to estimate the fractional part as

$$\hat{\nu} = \frac{1}{\pi} \arg \left\{ \sum_{k=\theta}^{\theta+N/2-1} r(k+N/2) r^*(k) \right\}$$

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Downlink Frequency acquisition (IV)

Fractional CFO estimate is compensated (counter- rotating the time domain Samples) and fed to DFT unit.
If $R_1(n)$ and $R_2(n)$ are the DFT of first and 2nd block, DFT outputs will be shifted from their correct position if $\eta \neq 0$ (remember frequency effects of CFO), i.e.,

$$R_1(n) = e^{j\varphi_1} H(|n-2\eta|_N) c_1(|n-2\eta|_N) + W_1(n)$$

$$R_2(n) = e^{j(\varphi_1 + 4\pi\eta N_T/N)} H(|n-2\eta|_N) c_2(|n-2\eta|_N) + W_2(n)$$

Downlink Frequency acquisition (V)

Neglecting noise terms, and defining $d(n) = c_2(n)/c_1(n)$, it is possible to conclude that, for *n* even

$$R_2(n) \cong e^{j4\pi\eta N_T/N} d(|n - 2\eta|_N) R_1(n)$$

Then, an estimate of η can be obtained by looking for the integer that maximizes

$$B(\tilde{\eta}) = \frac{\left|\sum_{n\in J} R_2(n) R_1^*(n) d(|n - 2\eta|_N)\right|}{\sum_{n\in J} |R_2(n)|^2}$$

where *J* is the set of indices for the even subcarriers.

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Downlink Frequency acquisition (VI)

Final CFO estimation of the S-C algorithm is given by

$$\hat{\epsilon} = \hat{\nu} + 2\hat{\eta}$$

Its MSE can be approximated by

$$MSE\{\hat{\epsilon}\} = \frac{2(SNR)^{-1}}{\pi^2 N}$$

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Downlink Frequency acquisition (VII)

M-M algorithm: extension of S-C algorithm, by considering a reference block composed by $Q \geq 2$ repetitive parts, each comprising $P = N/Q$ time domain samples. The estimated CFO is

$$\hat{\epsilon} = \frac{Q}{2\pi} \sum_{q=1}^{Q/2} \xi(q) \arg\{\Psi(q)\Psi(q-1)\}$$

$$\Psi(q) = \sum_{k=\theta}^{\theta+N-qP-1} r(k+qP)r^*(k), \quad q = 1, 2, \cdots, Q/2$$

$$\xi(q) = \frac{12(Q-q)(Q-q+1) - Q^2}{2Q(Q^2-1)}$$

Downlink Frequency acquisition (VIII)

If *Q* is designed such that CFO lie in $[-Q/2, Q/2]$, the M-M algorithm gives a CFO estimate that not requires a 2nd reference block.

The MSE of the estimate is given by

$$MSE\{\hat{\epsilon}\} = \frac{3(SNR)^{-1}}{2\pi^2 N(1-1/Q^2)}$$

lower than that of the S-C algorithm for *Q* > *2*.

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Downlink timing tracking

- Non negligible errors in the sampling clock frequency (due to clock oscillators) should result in a short–term variation of the timing error $\Delta\theta$ which must be tracked.

- Basic solution: associate $\Delta\theta$ as introduced by the channel rather than oscillator drift, that means to replace $\boldsymbol{h} = [h(0), h(1), \cdots, h(L-1)]^T$ by $\boldsymbol{h}'(\Delta\theta) = [h(\Delta\theta), h(1+\Delta\theta), \cdots, h(L-1+\Delta\theta)]^T$
  Then, channel estimates over different blocks are differently delayed.

  To track these fluctuations look for the delay of the first significant tap of the estimated CSI. The integer part of this delay is used to control DFT window, and the fractional part is compensated be the channel equalizer.

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Downlink timing tracking (II)

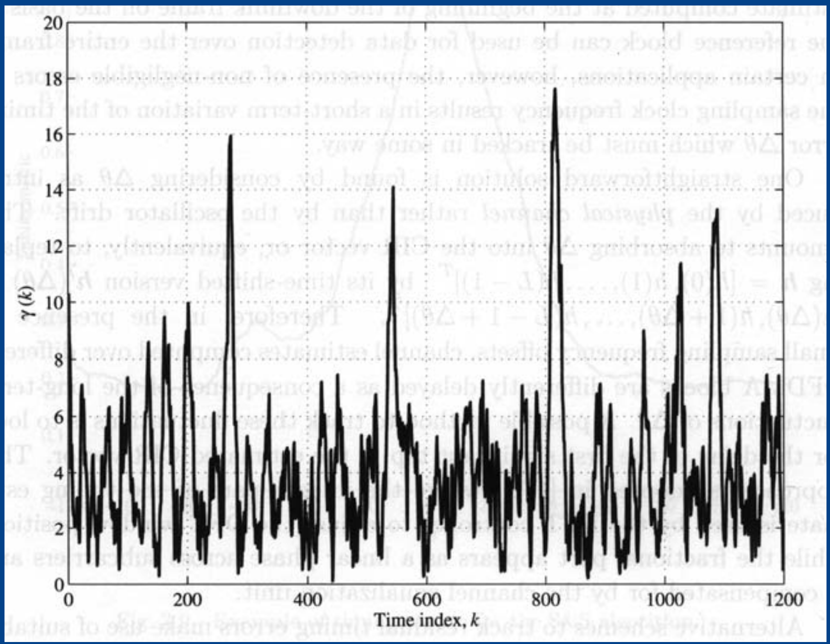
A CP-based timing tracking scheme: The following time metric is used

$$\gamma(k) = \sum_{k=0}^{N_g-1} r(k+q)r^*(k-q-N), \quad k \text{ current received sample}$$

Since the CP introduces periodicity due to the identically repeated *Ng* samples of each block, $\gamma(k)$ will exhibit peaks

Downlink timing tracking (III)

Example: N= 256, Ng=16. Rayleigh multipath channel with L=8 taps.



A remedy to have more robust results against interference and noise is to introduce the following smoother to the obtained estimate

$$\bar{\gamma}(k) = \alpha\bar{\gamma}(k - N_T)(1 - \alpha)\gamma(k)$$

where $0 < \alpha \leq 1$ must chosen to tradeoff between estimation accuracy and tracking capabilities.

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Downlink frequency tracking

CFO estimate obtained during the acquisition phase is used to adjust the LO to produce new received samples

$$r'(k) = r(k)e^{-j2\pi k\hat{\epsilon}/N}$$

Since $r'(k)$ may still be affected by a residual frequency error $\Delta\epsilon = \epsilon - \hat{\epsilon}$ and a non-negligible ICI will be present at DFT output.

In such conditions, tracking becomes necessary.

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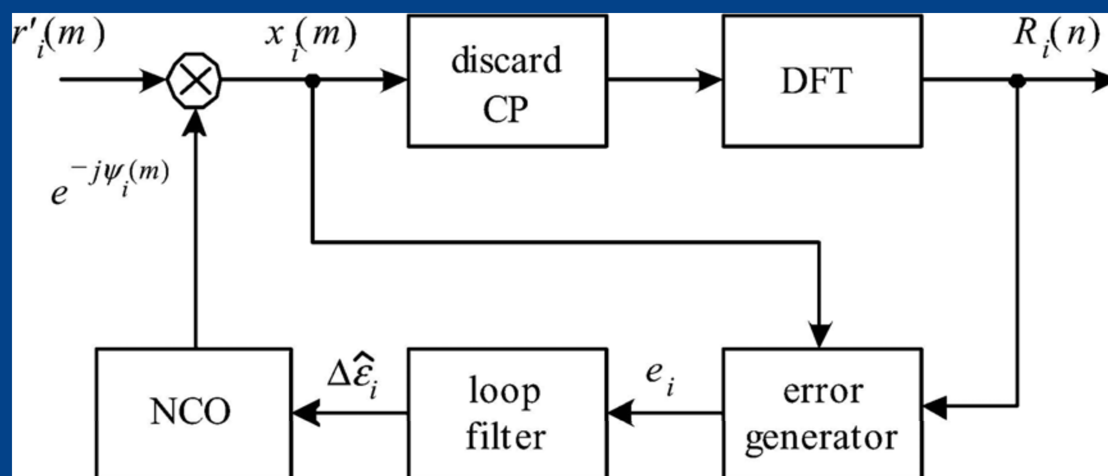
Downlink frequency tracking (II)

A general close-loop scheme for frequency tracking uses $r'(m)$ $(-N_g \leq m \leq N-1)$ and to obtain $R_i(n)$ and time or frequency strategies to define e_i , the error proportional to the residual CFO.

For each new received block

$$\Delta\hat{\epsilon}_{i+1} = \hat{\epsilon}_i + \alpha e_i$$



Downlink frequency tracking (III)

The corrective phase is computed recursively as

$$\psi_i(m) = \psi_i(m-1) + 2\pi\Delta\hat{\epsilon}/N, \ -N_g \leq m \leq N$$

where $\psi_i(-N_g - 1)$ is set equal to $\psi_{i-1}(N-1)$, to avoid any phase jump between the last and first samples of blocks (*i-1*) and *i*.

Depending on whether e_i is obtained using $R_i(n)$ or $x_i(m)$ it is possible to consider different solutions.

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Downlink frequency tracking (IV)

Time domain scheme: Uses the redundancy offered by the CP to obtain

$$e_i = \frac{1}{N_g} Im \left\{ \sum_{m=-N_g}^{-1} x_i(m+N) x_i^*(m) \right\}$$

A small perturbation analysis is useful to obtain an interpretation. Without any interference, the presence of a residual frequency offset leads to

$$x_i(n+N) \cong x_i(n) e^{j2\pi(\Delta\epsilon - \Delta\hat{\epsilon}_i)}, \; -N_g \leq m \leq -1$$

that means, $e_i \cong K \sin[2\pi(\Delta\epsilon - \Delta\hat{\epsilon}_i)]$. Note that the sign of $(\Delta\epsilon - \Delta\hat{\epsilon}_i)$ defines that of e_i, and then a decrement or increment of $\Delta\hat{\epsilon}_{i+1}$, where the equilibrium point is $\Delta\hat{\epsilon}_{i+1} = \Delta\hat{\epsilon}_i$.

Downlink frequency tracking (V)

Frequency domain approach: a basic scheme, based on ML, uses

$$e_i = Re\left\{\sum_{n\in\mathcal{I}} R_i^*(n)[R_i(n+1) - R_i(n-1)]\right\}$$

An improved variant considers

$$e_i = Re\left\{\sum_{n\in\mathcal{I}} \frac{R_i^*(n)[R_i(n+1) - R_i(n-1)]}{1+\beta|R_i(n)|^2}\right\}$$

where β depends on the operating SNR.

Conclusions

- In a general time varying frame context (i.e., time varying during the frame length), synchronization is divided in acquisition and tracking.
- Timing acquisition and tracking can be performed based on basic cross-correlation techniques.
- Carrier frequency offset estimation methods analyzed (all use training sequences) have different range (if related to subcarrier separation), with some advantage for those that not require the estimation of the CFO integer part.