Initial ranging estimation

- Signal model and system description
- Estimation of the CFOs
- Estimation of the timing delays
- Performance

Initial ranging estimation

Initial ranging process:

- Each ranging subscriber station (RSS) computes frequency and timing estimates using a downlink control channel.
- Each RSS transmits a randomly chosen code over a ranging time-slot.
- After identifying colliding codes and extracting timing and power information (uplink signals arrive at the BS at different time instants and several users may collide over a same timeslot), BS broadcast a response message indicating codes detected and giving instructions for timing and power adjustment.

Main functions of BS in ranging process: multiuser code detection and multiuser timing/power estimation.

Initial ranging estimation

- Methods to accomplish these tasks consist of a long pseudo-noise (PN) code transmitted by each RSS over all available ranging subcarriers.
- Code detection and timing recovery is then accomplished using correlations computed in either the frequency or time domains.
- This requires huge computational complexity since one correlation must be evaluated for each possible ranging code and timing offset.
- Moreover, in the presence of multipath distortions ranging subcarriers are subject to different attenuations and phase shifts, leading to a loss of the code orthogonality, i.e. MAI.

Initial ranging estimation

Better performance on frequency selective channels can be obtained using the following scheme (FLM)

- Each RSS selects a ranging subchannel composed of a specified number of subcarriers (not necessarily adjacent), and transmits a randomly chosen code over adjacent OFDMA symbols.
- Spreading is performed in the time domain (same code is transmitted in parallel over all selected subcarriers). In a perfectly frequency synchronized scenario, codes transmitted on different subcarriers remain disjoint at the receiver (i.e., if channel keeps constant during the overall ranging period, codes received over the same subcarrier are still orthogonal and can easily be separated at the BS).
- After multiuser code detection, timing information is eventually acquired through an iterative procedure (autocorrelation properties induced by CP).

Initial ranging estimation

Drawbacks of FLM

- In spite of the improved robustness against channel distortions, spreading across adjacent symbols increases the sensitivity of the system to residual CFOs.

- Phase rotations of the codes may become significant if the ranging period spans several adjacent symbols. In such a case, the received ranging signals are no longer orthogonal and CFO compensation is necessary.

Initial ranging estimation

Following a similar orthogonal signal design, we are going to study a method that uses a multistage approach:

- The number of active codes is first determined by resorting to the minimum description length (MDL) principle.
- The multiple signal classification (MUSIC) algorithm is next employed for code identification and CFO estimation.
- Frequency estimates are then used in the third step, where timing and power level estimation is accomplished in an ad-hoc fashion.

System description and signal model

N : subcarriers with frequency spacing Δf and indices in $\mathcal{I} = \{0,1,\ldots,N-1\}$.

N_0 : null subcarriers, placed at both edges of the signal spectrum.

R: number of ranging subchannels, each divided into *Q* subbands uniformly spaced over the signal bandwidth at a distance $(N_U/Q)\Delta f$ from each other, where $N_U = N - 2N_0$ is the number of modulated subcarriers.

Subband (tile): composed by a set of *V* adjacent subcarriers.

M the number of OFDM symbols in a ranging time-slot (a power of two).

System description and signal model

Subcarrier indices in the *q*th tile of the *r*th subchannel are collected in

$$\mathcal{I}_q^{(r)} = \left\{ i_{q,\nu}^{(r)} \right\}_{\nu=0}^{V-1} \qquad i_{q,\nu}^{(r)} = \frac{qN_U}{Q} + \frac{rN_U}{QR} + N_0 + \nu$$

The *r*th subchannel is thus composed of subcarriers with indices in

$$\mathcal{I}^{(r)} = \cup_{q=0}^{Q-1} \mathcal{I}_q^{(r)}$$

A total of $N_R = QVR$ ranging subcarriers are available with indices in

$$\mathcal{I}_R = \cup_{q=0}^{R-1} \mathcal{I}^{(r)}$$

The remaining $N_D = N_U - N_R$ subcarriers are used for data transmission and are assigned to DSSs which have successfully completed their IR process at an earlier stage.

System description and signal model

Proposed ranging process

1) Each RSS selects one of the *R* available ranging subchannels according to some specified criterion (i.e., depending on channel conditions, in low mobility).

2) RSS transmits a randomly chosen code of length *M* during the ranging time-slot. Such a code is transmitted in parallel over all subcarriers belonging to the selected subchannel and is taken from an orthogonal set (e.g., a Walsh-Hadamard or a Fourier basis set)

$$\mathcal{C} = \{\mathbf{c}_1, \mathbf{c}_2, \cdots, \mathbf{c}_M\}$$

$$\mathbf{c}_k = [c_k(0), c_k(1), \cdots, c_k(M-1)]^T, \ |c_k(m)| = 1 \ \ 0 \le m \le M-1$$

3) With the received uplink signals, the BS determines which codes are employed and extracts frequency, timing and power information. It also detects possible collisions between RSSs that use the same code and ranging subchannel.

4) Once the above operations have been successfully completed, the BS will broadcast a response message by which the detected RSSs can adjust their synchronization parameters.

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System description and signal model

We concentrate on the rth subchannel and assume that it has been selected by $K^{(r)}$ RSSs (subchannel index (r) is dropped henceforth).

The waveform transmitted by the kth RSS propagates through a multipath channel with impulse response $\boldsymbol{h}_k = [h_k(0), h_k(1), \cdots, h_k(L_k - 1)]^T$. As L_k is usually unknown, we replace $\boldsymbol{h}_k$ by an L-dimensional vector $\boldsymbol{h}'_k = [\boldsymbol{h}_k^T, 0, \cdots, 0]^T$, where $L \geq \max_k\{L_k\}$.

System description and signal model

At the BS, the received samples are not synchronized. Then, θ_k and ϵ_k characterize timing and CFO error of user *k*.

Due to initial downlink sync, during IR the CFOs are only induced by Doppler shifts and/or downlink estimation errors and, in consequence, they will be quite small.

Timing errors depend on BS – RSS distances, $\theta_{max} = 2R_c/(cT_s)$. To counteract them we assume long CPs, i.e., $N_G \geq \theta_{max} + L$, *the quasi-synchronous scenario* (adopted during IR, but not with the CP of data symbols, i.e., avoid overhead. Then, accurate timing estimates must be obtained during IR to avoid IBI over the data section of the frame).

System description and signal model

Signal model:

- $Y_m(i_{q,\nu})$ is the DFT output over the $i_{q,\nu}$ subcarrier of the *m*th OFDM symbol. Assuming the DSSs are perfectly synchronized to BS, their signals do not contribute to $Y_m(i_{q,\nu})$. However, uncompensated CFOs destroy orthogonality among ranging signals and gives rise to ICI.

- On the other hand, since CFOs are a small fraction of the subcarrier spacing, the demodulated signals incur negligible phase rotations over one OFDM symbol and the resulting ICI can be neglected. Then

$$Y_m(i_{q,\nu}) \cong \sum_{k=1}^{K} c_k(m) e^{j2\pi m \epsilon_k N_T/N} S_k(\theta_k, i_{q,\nu}) + n_m(i_{q,\nu})$$

System description and signal model

where
$$S_k(\theta_k, i_{q,\nu}) = e^{j2\pi\theta_k\epsilon_k/N} \quad H_k(i_{q,\nu})$$

and
$$H_k(i_{q,\nu}) = \sum_{\ell=0}^{L-1} h_k(\ell) e^{-j2\pi\ell i_{q,\nu}/N}$$

is the *k*th channel frequency response. Also, the power from the *k*th RSS over the ranging subcarriers is

$$P_k = \frac{1}{QV} \sum_{q=0}^{Q-1} \sum_{\nu=0}^{V-1} |S_k(\theta_k, i_{q,\nu})|^2$$

Code detection and CFO estimation

Collecting DFT outputs across the ranging time-slot, we obtain

$$\boldsymbol{Y}(i_{q,\nu}) = \sum_{k=1}^{K} \boldsymbol{S}_k(\theta_k, i_{q,\nu}) \boldsymbol{\Gamma}(\epsilon_k) \boldsymbol{c}_k + \boldsymbol{n}(i_{q,\nu})$$

$$\boldsymbol{\Gamma}(\epsilon_k) = \mathrm{diag}\{e^{j2\pi m \epsilon_k N_T / N};\ m = 0, 1, \cdots, M-1\}$$

Since this is the superposition of frequency rotated codes, embedded in WGN, this model has the same structure as measurements of multiple uncorrelated sources from array sensors, i.e., we can use subspace-based methods to obtain active codes and CFOs.

Code detection and CFO estimation

Number of active codes: can be obtained, based on information-theoretic criteria, using the eigenvalue decomposition of $\boldsymbol{R_Y} = E\{\boldsymbol{Y}(\boldsymbol{i}_{q,\nu})\boldsymbol{Y}^H(\boldsymbol{i}_{q,\nu})\}$ or its estimation

$$\hat{\boldsymbol{R}}_{\boldsymbol{Y}} = \frac{1}{QV}\sum_{\nu=0}^{V-1}\sum_{q=0}^{Q-1}\boldsymbol{Y}(\boldsymbol{i}_{q,\nu})\boldsymbol{Y}^H(\boldsymbol{i}_{q,\nu})$$

Following MDL, $\hat{K}$ is obtained from

$$\hat{K} = \arg\min_{\tilde{K}}\{\mathcal{F}(\tilde{K})\}$$

$$\mathcal{F}(\tilde{K}) = \frac{1}{2}\tilde{K}(2M-\tilde{K})\ln(QV) - QV(M-\tilde{K})\ln\rho(\tilde{K}) \qquad \rho(\tilde{K}) = \frac{\left(\prod_{i=\tilde{K}+1}^{M}\hat{\lambda}_i\right)^{\frac{1}{M-\tilde{K}}}}{\frac{1}{M-\tilde{K}}\sum_{i=\tilde{K}+1}^{M}\hat{\lambda}_i}$$

Code detection and CFO estimation

CFO estimation and code detection

- Based on the subspace interpretation, the signal subspace $\mathcal{S}_s$ *spanned by the rotated* codes $\{\boldsymbol{\Gamma}(\epsilon_k)\boldsymbol{c}_k\}_{k=1}^{K}$, is orthogonal to the noise subspace $\mathcal{S}_n$

- *We assume that* the number of active codes has been correctly estimated and denote by $\{\hat{\boldsymbol{u}}_1, \hat{\boldsymbol{u}}_2, \cdots, \hat{\boldsymbol{u}}_M\}$ the eigenvectors of $\hat{\boldsymbol{R}}_{\boldsymbol{Y}}$ corresponding to the ordered eigenvalues $\hat{\lambda}_1 \geq \hat{\lambda}_2 \geq \cdots \geq \hat{\lambda}_M$.

- MUSIC algorithm relies on the fact that $\{\hat{\boldsymbol{u}}_{K+1}, \hat{\boldsymbol{u}}_{K+2}, \cdots, \hat{\boldsymbol{u}}_M\}$ associated form an estimated basis of $\mathcal{S}_n$ and are *approximately* orthogonal to the signal space.

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Code detection and CFO estimation

- An estimate of ϵ_k is thus obtained by minimizing the projection of $\boldsymbol{\Gamma}(\tilde{\epsilon}_k)\boldsymbol{c}_k$ onto the subspace spanned by the columns of $\hat{\boldsymbol{U}}_n = [\hat{\boldsymbol{u}}_{K+1}, \hat{\boldsymbol{u}}_{K+2}, \cdots, \hat{\boldsymbol{u}}_M]$ i.e.

$$\hat{\epsilon}_k = \arg\max_{\tilde{\epsilon}}\{\Psi_k(\tilde{\epsilon}_k)\} \qquad \Psi_k(\tilde{\epsilon}) = \frac{1}{\left\| \hat{\boldsymbol{U}}_n^H \boldsymbol{\Gamma}(\tilde{\epsilon}_k)\boldsymbol{c}_k \right\|^2}$$

- Since CFOs need to be estimated for each active code, then minimization requires to consider all codes.

- The criterion to declare *active codes* is to look for the *K* largest values of

$$\{\Psi_k(\hat{\epsilon}_{u_k})\}_{k=1}^{K}$$

Code detection and CFO estimation

Phantom RSS:

Let $K \leq M - 1$, as required for the MUSIC estimator, and assume that two or more RSSs share the same ranging code over chosen subchannel. MDL provides the exact number of active RSSs, but MCD is not capable of identifying the corresponding codes.

As an example, let $M = 4$ and $K = 3$ with one RSS employing code ***c1*** and the other two RSSs using **c2**. In this situation, it is likely that the MCD declares as active **c1** and **c2** plus a third code (**c3** or **c4)** which is actually turned off and corresponds to a *phantom* RSS.

In a similar way, the MFE will provide three CFO estimates, two of which are associated with active users while the remaining one corresponds to the phantom RSS.

Code detection and CFO estimation

Alternatives:

In multiple frequency estimation problems the ESPRIT represents a valid alternative to the MUSIC.

The main advantage of ESPRIT is that it provides estimates of the signal parameters in closed-form without requiring any time consuming grid-search.

A basic assumption behind this technique is the rotational invariance property of the observation vectors, which is guaranteed in the presence of complex exponentials in noise. Unfortunately, in general, the rotated codes do not satisfy the invariance property.

Code detection and CFO estimation

A key issue is the maximum CFO that the MFE can handle. An available result assume that the ranging codes belong to the Fourier basis of order *M*, given by

$$c_k(m) = e^{j2\pi m(k-1)/M}, \quad 0 \leq m \leq M-1$$

Then, identifiability of the rotated codes is guaranteed as long as the normalized CFOs are smaller than $N/(2MN_T)$ in magnitude.

Timing delay and power level estimation

We consider rewrite the signal model in a more compact form as

$$\boldsymbol{Y}(i_{q,\nu}) = \boldsymbol{C}(\boldsymbol{\epsilon})\boldsymbol{S}(\boldsymbol{\theta}, i_{q,\nu}) + \boldsymbol{n}(i_{q,\nu})$$

$$\boldsymbol{\theta} = [\theta_1, \theta_2, \cdots, \theta_K]^T \qquad \boldsymbol{\epsilon} = [\epsilon_1, \epsilon_2, \cdots, \epsilon_K]^T$$

$$\boldsymbol{C}(\boldsymbol{\epsilon}) = [\boldsymbol{\Gamma}(\epsilon_1)\boldsymbol{c}_1 \;\; \boldsymbol{\Gamma}(\epsilon_2)\boldsymbol{c}_2 \;\; \cdots \boldsymbol{\Gamma}(\epsilon_K)\boldsymbol{c}_K]$$

Timing delay and power level estimation

- Then, assuming $\hat{\boldsymbol{K}} = \boldsymbol{K}$ and $\hat{\epsilon}_k \approx \epsilon_k$, the ML estimate of $\boldsymbol{S}(\boldsymbol{\theta}, i_{q,\nu})$ is given by

$$\hat{\boldsymbol{S}}(i_{q,\nu}) = [\hat{\boldsymbol{C}}(\hat{\epsilon})^H \hat{\boldsymbol{C}}(\hat{\epsilon})]^{-1} \hat{\boldsymbol{C}}^H(\hat{\epsilon}) \boldsymbol{Y}(i_{q,\nu})$$

that replaced in previous equation gives

$$\hat{\boldsymbol{S}}(i_{q,\nu}) = \boldsymbol{S}(i_{q,\nu}) + \boldsymbol{\psi}(i_{q,\nu})$$

where $\boldsymbol{\psi}(i_{q,\nu}) = [\psi_1(i_{q,\nu}), \psi_2(i_{q,\nu}) \cdots, \psi_K(i_{q,\nu})]^T$, is a zero mean disturbance vector with covariance matrix $\sigma^2[\hat{\boldsymbol{C}}(\hat{\epsilon})^H \hat{\boldsymbol{C}}(\hat{\epsilon})]^{-1}$, that can be written in scalar form as

$$\hat{S}_k(i_{q,\nu}) = e^{-j2\pi\theta_k i_{q,\nu}/N} H_k(i_{q,\nu}) + \psi_k(i_{q,\nu}), \quad 1 \leq k \leq K$$

Timing delay and power level estimation

- Then, assuming perfect CFO and code estimation, $\hat{S}(i_{q,\nu})$ has only contribution of *k*th RSS timing.

- An estimation of θ_k can be obtained considering that $H_k(i_{q,\nu}-1) \cong H_k(i_{q,\nu})$ then,

$$\hat{S}_k(i_{q,\nu}-1)\hat{S}_k^*(i_{q,\nu}) \cong |H_k(i_{q,\nu})|^2 e^{j2\pi\theta_k/N}$$

that results in

$$\hat{\theta}_k = \frac{N}{2\pi}\arg\left\{\sum_{q=0}^{Q-1}\sum_{\nu=1}^{V-1}\hat{S}_k(i_{q,\nu}-1)\hat{S}_k^*(i_{q,\nu})\right\}$$

Timing delay and power level estimation

Power level estimation:

Since $\hat{S}(i_{q,\nu})$ is an unbiased estimator of $S(i_{q,\nu})$, then an unbiased estimator of $|S(i_{q,\nu})|^2$ is given by $|\hat{S}(i_{q,\nu})|^2 - \sigma_k^2$.

Then, the following power estimator can be obtained

$$\hat{P}_k = \frac{1}{QV} \sum_{q=0}^{Q-1} \sum_{\nu=0}^{V-1} |\hat{S}_k(i_{q,\nu} - 1)|^2 - \hat{\sigma}_k^2$$

$$\hat{\sigma}_k^2 = \hat{\sigma}^2 \left\{ [\hat{C}(\hat{\epsilon})^H \hat{C}(\hat{\epsilon})]^{-1} \right\}_{k,k}$$

Collision detector

- We neglected possible collisions between RSSs that choose the same ranging opportunity. This implies that each subchannel is accessed by no more than $M - 1$ *RSSs* employing different codes.

- One critical situation is the presence of pairs of phantom RSSs. If the undetected RSS employs the same code of a detected RSS, the former will adjust its transmission parameters according to the response message transmitted to the latter. Such adjustment may have detrimental effects as it is based on incorrect synch information. (presence of a phantom RSS is not a big problem as the code associated to the corresponding response message will not recognized by any of the active RSSs).

- The above discussion indicates that collision events must be detected to avoid that incorrect synch information be transmitted to the active RSSs.

Collision detector

- Based on the signal model the following difference will have small value for perfect code detection

$$\Delta \boldsymbol{Y}(i_{q,\nu}) = \boldsymbol{Y}(i_{q,\nu}) - \hat{\boldsymbol{C}}(\hat{\epsilon})\hat{\boldsymbol{S}}(i_{q,\nu})$$

- In fact, it can be shown that

$$E\{\|\Delta \boldsymbol{Y}(i_{q,\nu})\|^2\} = \sigma^2(M - \hat{K}) + \delta(\hat{\boldsymbol{\epsilon}}, \boldsymbol{\epsilon}, i_{q,\nu})$$

$$\delta(\hat{\boldsymbol{\epsilon}}, \boldsymbol{\epsilon}, i_{q,\nu}) = \begin{cases} 0 & \text{if } \hat{K} = K \text{ and } \hat{\boldsymbol{\epsilon}} = \boldsymbol{\epsilon} \\ \|\hat{\boldsymbol{Z}}^H(\hat{\boldsymbol{\epsilon}})\boldsymbol{C}(\boldsymbol{\epsilon})\boldsymbol{S}(i_{q,\nu})\|^2 & \text{otherwise} \end{cases}$$

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Collision detector

A suitable estimator of δ, defined as collision metric, is

$$\hat{\delta} = \frac{1}{QV} \sum_{q=0}^{Q-1} \sum_{\nu=0}^{V-1} \|\Delta Y(i_{q,\nu})\|^2\} - \hat{\sigma}^2(M - \hat{K})$$

If $\hat{\delta}$ exceeds a specified threshold η a collision is declared. BS does not send any response message to users on the considered subchannel . The RSSs that do not find their information repeat the ranging process in the next frame using a different ranging opportunity.

If $\hat{\delta} < \eta$, BS sends a response message for all detected codes in the considered subchannel.

η must be designed to achieve a reasonable trade-off between the probability of declaring a collision when in fact it is not present (false alarm) and the probability of not detecting a collision when in fact it is present (mis-detection).

Initial ranging performance

Example: WiMax comparisons with FLM

- *N= 1024*, N_o = *80*.
- Ranging: N_R = *144* subcarriers, grouped in *R=18* subchannels with *Q=4* tiles (with *V*=2 subcarriers) spaced a distance $(N - N_o)/Q$ = 216. Time slot *M=4* symbols. Number of ranging opportunities per frame: $R(M-1)$ = *54*.
- Ranging codes: Fourier basis order 4.
- Data: $(N - N_o) - N_R$ = *720* subcarriers grouped in *15* data subchannels (QPSK).
- Channel: L=14 (max), coefficients circularly symmetric Gaussian and exponential power profile.
- Cell radius: 1.5 km (θ_{max} = 114 samples). N_G= 128 (ranging CP)
- CFOs: uniformly distributed $-\epsilon_{max} \leq \epsilon_k \leq \epsilon_{max}$ ($\epsilon_{max} \leq$ 0.1)

Initial ranging performance

- We consider a static scenario where channel is generated at each run and kept fixed over an entire time-slot.
- All RSSs attempt their ranging simultaneously at the first time-slot choosing different ranging opportunities.
- Comparisons are made with FLM with same number of ranging subcarriers, ranging subchannels, data subchannels, and the same transmitted energy from each user.
- The same number K of RSSs is present in *each* ranging subchannels. This implies that a total of $K R$ RSSs are simultaneously active in the system.
- Note that letting $K = 3$ in our ranging scheme corresponds to a fully-loaded system where all ranging opportunities are employed.

Initial ranging performance

Multiuser code detection

Mis-detection probability: Deteriorates when increasing *K* (due to reduction of noise subspace dimension degrades accuracy MCD).

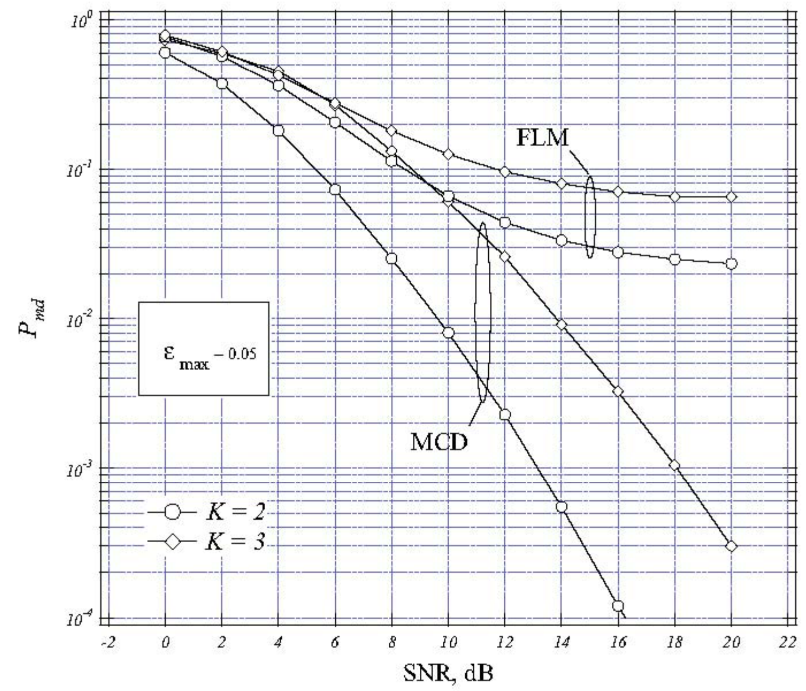

Fig. 1. Mis-detection probability vs. SNR with $K = 2$ or 3 and $\varepsilon_{\max} = 0.05$.

Initial ranging performance

Multiuser code detection

- False alarm probability: MCD outperform FLM for SNR > 6 dB.

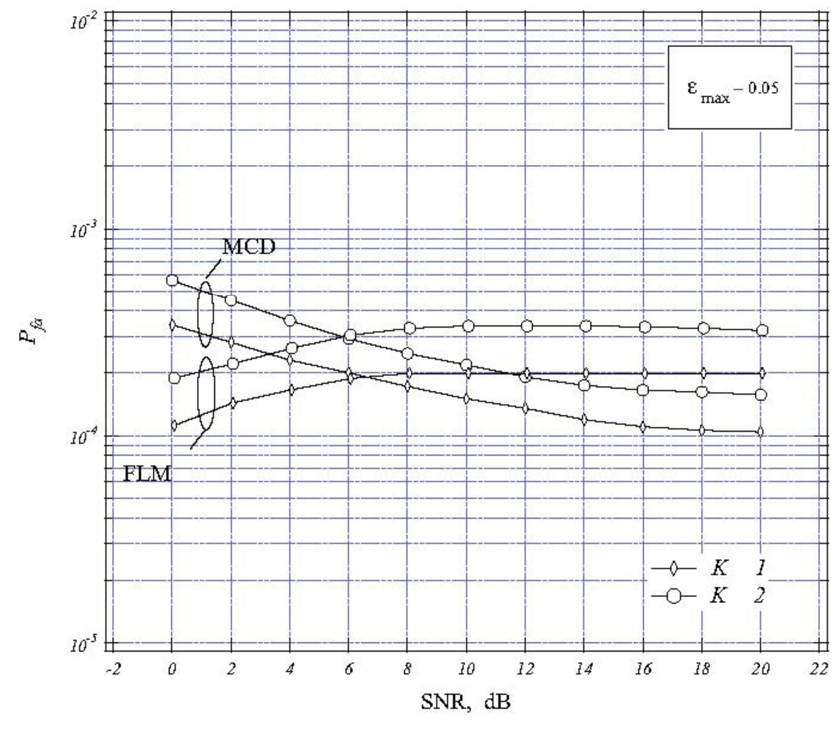

Fig. 2. False alarm probability vs. SNR with $K = 1$ or 2 and $\varepsilon_{\max} = 0.05$.

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Initial ranging performance

Frequency estimation

- Accuracy is maintained even for increased number of users.

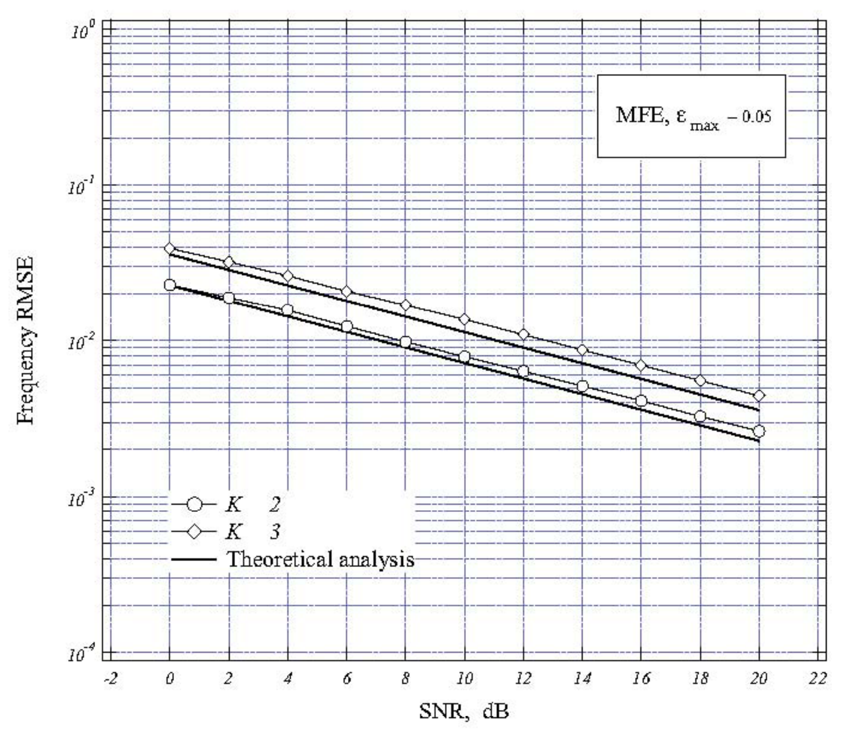

Fig. 3. Frequency RMSE vs. SNR with $K = 2$ or 3 and $\varepsilon_{\max} = 0.05$.

Initial ranging performance

Frequency estimation

Accuracy is only marginally affected by ϵ_{max}

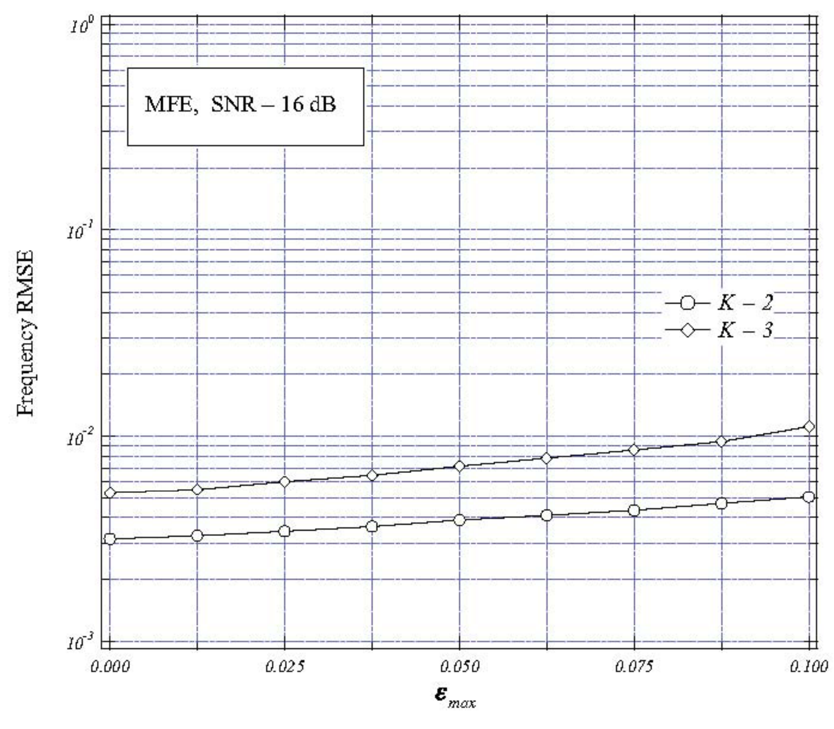

Fig. 4. Frequency RMSE vs. $\varepsilon_{\max}$ with K = 2 or 3 and SNR= 16 dB.

Initial ranging performance

Timing recovery

Performance assessed by measuring the probability of a timing error event, i.e., the timing error is larger than 0 or smaller than $L - N_{GD} - 1$ (N_{GD} =48, CP length during data transmission period).

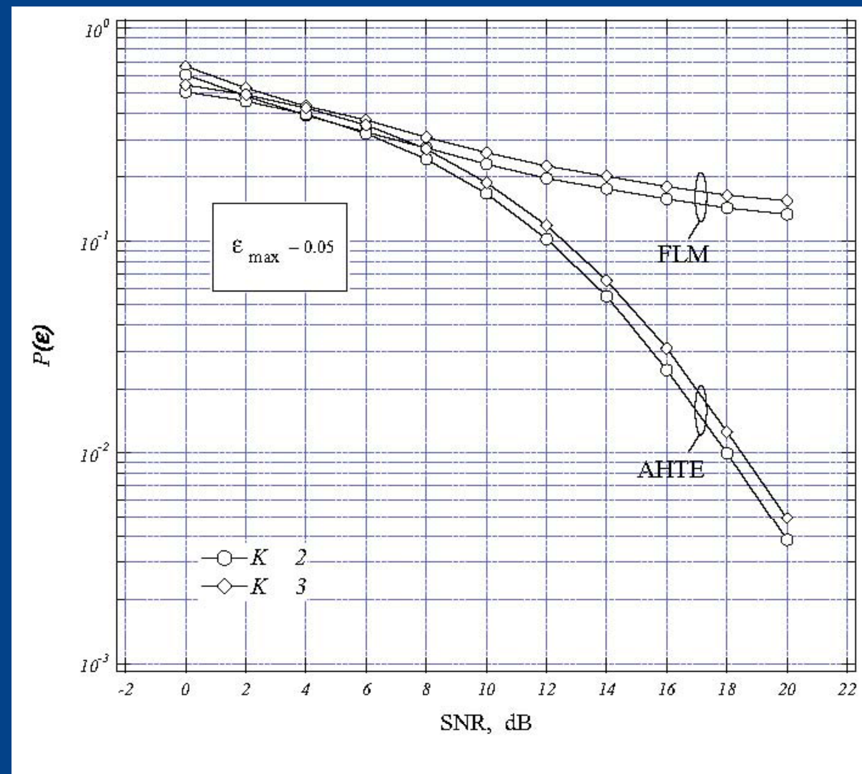

Fig. 5. $P(\epsilon)$ vs. SNR with K = 2 or 3 and $\varepsilon_{\max} = 0.05$.

Initial ranging performance

Timing recovery

Comparison in terms of sensitivity to CFOs.

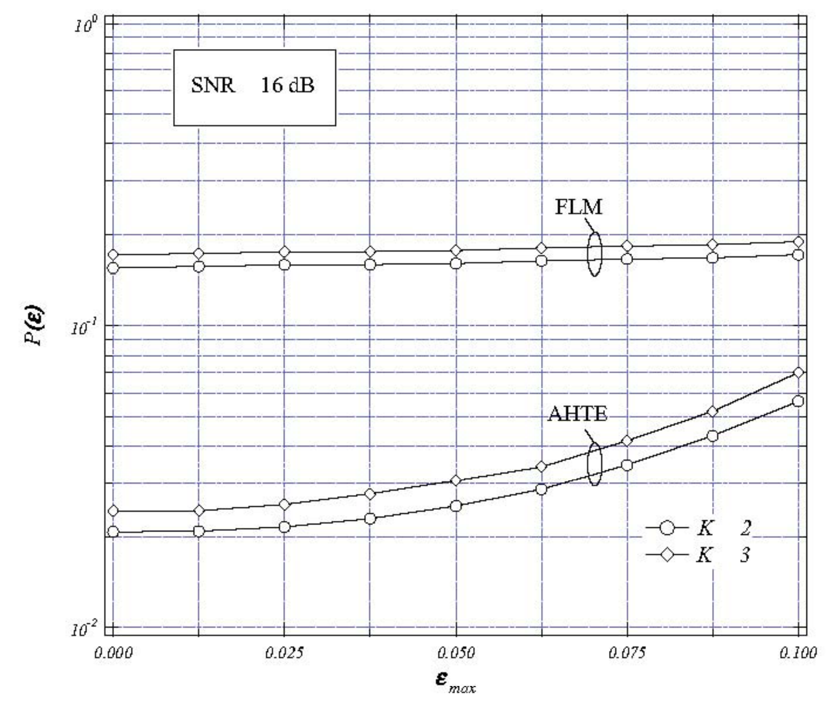

Fig. 6. $P(\epsilon)$ vs. $\varepsilon_{\max}$ with $K = 2$ or 3 and SNR= 16 dB.

Initial ranging performance

Power estimation

AHPE outperforms FLM for high SNR.

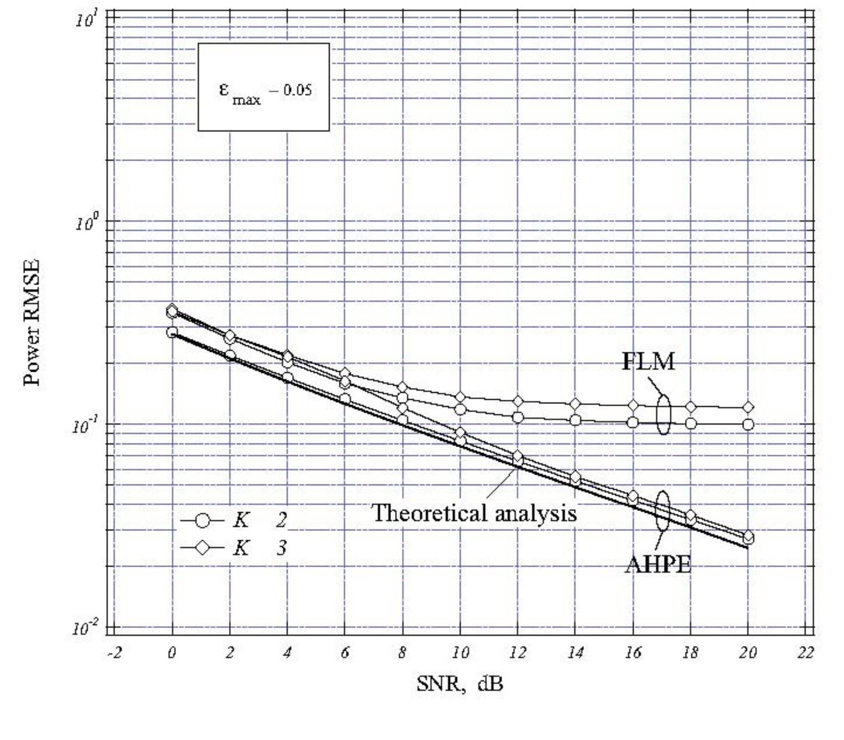

Fig. 7. RMSE of the power estimates vs. SNR with $K = 2$ or 3 and $\varepsilon_{max} = 0.05$.

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Initial ranging performance

Impact of channel variations

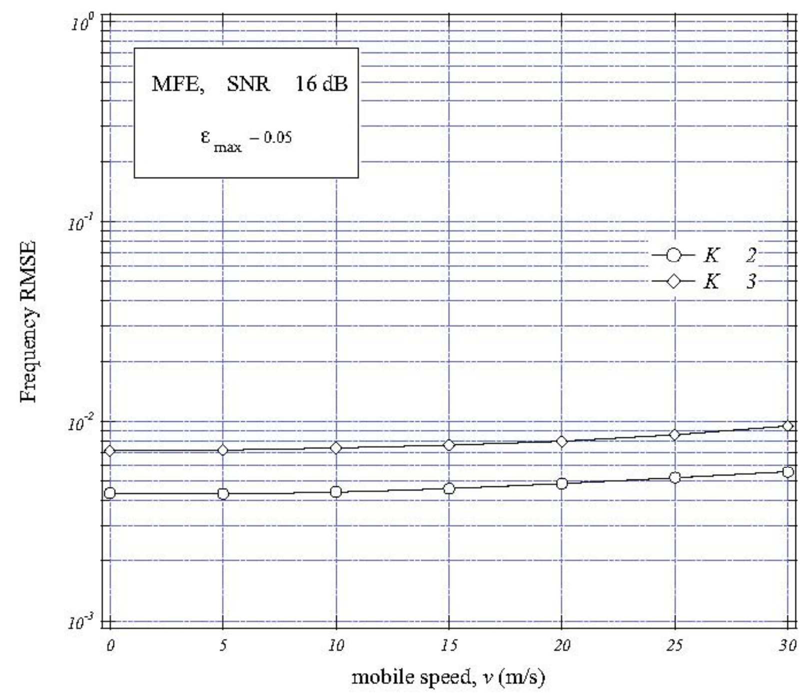

Fig. 8. Frequency RMSE vs. v with K = 2 or 3 and SNR = 16 dB.

Conclusions

- Initial ranging process is critical in terms of timing and frequency synchronization in WiMAX.
- An alternative to improve the basic methods to estimate timing synch is to design ranging signals separated in frequency groups.
- A MUSIC-based scheme provides robust CFO and timing estimates.
- Main drawback of the scheme studied is computational complexity.