A Novel Post-FFT OFDM Receiver Beamforming in the Presence of Downconverter Impairments

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Abstract— **In this paper, two non-desired effects, interference and in-phase/quadrature (I/Q) imbalance, are jointly compensated at the receiver side of an Orthogonal Frequency Division Multiplexing (OFDM) system. Taking information of the known pilots transmitted within the symbol, a Post-FFT receiver beamforming based on an NLMS algorithm is proposed to adapt the beamforming weights to reject interference signals and to compensate I/Q imbalance. Our simulation results show that the proposed Post-FFT receiver beamforming compensates I/Q imbalance without degrading the interference rejection of the array.**

Keywords— **Beamforming, I/Q imbalance, Adaptive Antennas Array, Spatial Processing, Interference Cancellation.**

1 INTRODUCTION

Orthogonal Frequency Division Multiplexing (OFDM) is a robust technique considering severe multipath channels in high-speed data transmission [1]. Emerging communication technologies require high spectral efficiency and robustness against time dispersive channels. For this reason, OFDM has been adopted in the main wireless communication standards [2, 3, 4].

A key OFDM advantage is the fact that the use of a cyclic prefix (CP) eliminates the Inter-Symbol Interference (ISI) allowing a low complexity single-tap equalization. On the other hand, multiple antennas at the receiver side offer many solutions for various problems that may affect the system performance. For instance, multiple antennas together with spatial processing techniques give the system the capacity to suppress interference signals. The information carried in the pilot symbols is used to estimate the optimum weights for the beamforming, and spatial signal processing at the receiver can be applied in either time or frequency domain.

Time-domain processing has a lower complexity because only one Fast Fourier Transform (FFT) operation is required. Despite that frequency-domain processing requires an FFT operation per each antenna, it gives better performance in most of the circumstances [5, 6]. Array processing in the time-domain is normally called Pre-FFT, while Post-FFT is the name given for frequency-domain array processing.

In this paper we apply our algorithm in the Post-FFT scheme. The in-phase/quadrature (I/Q) imbalance in the down conversion introduces the undesired signal image interference, which limits the demodulation performance to an error floor [7]. Even when a considerable amount of research has been performed related to these two spatial processing techniques, there are no specific results available related to the design of beamforming algorithms for systems affected by receiver I/Q imbalance. In this paper we propose a beamforming technique that takes into account the I/Q imbalance and the compensation technique associated, achieving good results.

The paper is organized as follows. The system model and beamforming methods are introduced in Section 2. Section 3 presents the beamforming technique proposed. In Section 4, simulations results are presented, followed by conclusions in Section 5.

2 SYSTEM MODEL AND BEAMFORMING METHODS

2.1 System Model

A generic OFDM symbol can be written as:

$$s[n] = \frac{1}{\sqrt{N}} \sum_{i=0}^{N-1} S_i \exp\{j2\pi in/N\} \quad (1)$$

where N is the number of data samples in the OFDM symbol without the cyclic prefix and n denotes the sample within the symbol. Before the symbol is transmitted, the cyclic prefix must be added. We consider the transmission using a Rayleigh channel affected by additive white Gaussian noise (AWGN). The impulse response of the multipath channel can be modeled as

$$h(t) = \sum_{i=1}^{K} \alpha_i \delta(t - \tau_i) \quad (2)$$

where K is the number of multipaths, τ_i is the delay of arrival of the ith path and α_i is the complex factor that affects each individual path. In this work, the channel taps are assumed constant within one OFDM symbol (block fading assumption).

We also consider an Uniform Linear Array (ULA), with M omnidirectional antennas equally spaced $\lambda/2$, with λ the wavelength of the signal carrier. To model this, each received ray is multiplied by its corresponding steering vector which takes into account the direction of arrival (DoA) of each ray. Considering a ray arriving at an angle θ, the signal is multiplied by $\exp(\pi j\ m \sin(\theta))$, with $m = 0, ..., M-1$. So, the steering vector $\mathbf{b}(\cdot)$ is defined by

$$\mathbf{b}(\theta) = [1\ \exp(\pi j \sin(\theta))\ ...\ \exp(\pi j (M-1)\sin(\theta))]^T \tag{3}$$

The signal received by the ULA can be represented by a $N_T \times M$ matrix, where N_T is the OFDM frame with the cyclic prefix, after discarding the cyclic prefix and without interference we get a $N \times M$ matrix given by

$$\mathbf{X}_d = \sum_{i=1}^{K} \mathbf{b}(\theta_i)\mathbf{r}_i^T + \mho \tag{4}$$

where $\mathbf{r}_i$ are column vectors of the multipath signals and $\mho$ represents the AWGN, in which every element of $\mho$ are independent Gaussian random variables. If we take into account interference signals, the received signal can be written as

$$\mathbf{X} = \mathbf{X}_d + \sum_{l=0}^{L-1} \mathbf{I}_l \tag{5}$$

where $\mathbf{I}_l$ is calculated in the same way as $\mathbf{X}_d$ and L is the number of interfering users. In the down-conversion, the signal is affected by I/Q imbalance. To take this into account the received signal is modeled as in the following expression ($*$ represents the complex conjugate)

$$\mathbf{X}_{IQ} = \eta\mathbf{X} + \nu\mathbf{X}^* \tag{6}$$

The distortion parameters η and ν are related to the amplitude and phase imbalances between the I and Q branches of the down-conversion through a simplified model, as follows

$$\mu = \cos(\beta/2) + j\alpha\sin(\beta/2) \tag{7}$$

$$\nu = \alpha\cos(\beta/2) + j\sin(\beta/2) \tag{8}$$

where α and β are, respectively, the phase and amplitude imbalance between the I and Q branches.

2.2 Beamforming Methods

After the down-conversion and the analog to digital stage, the signals must be translated to frequency-domain by the use of the FFT. How many FFTs are required depends on the chosen smart antenna architecture. Frequency-domain signals will be noted by a tilde (˜) over the symbol.

2.2.1 Pre-FFT

In the Pre-FFT beamforming the signal of each antenna affected by the I/Q imbalance is multiplied by its corresponding Pre-FFT weight and added together to build the signal $\mathbf{y}$ which is then translated to frequency-domain by an FFT operation. Specifically, this process can be written as

$$\mathbf{y} = \mathbf{w}_{pre}^H \mathbf{X}_{IQ}^T \tag{9}$$

$$\mathbf{w}_{pre} = [w_{pre}^1\ w_{pre}^2\ ...\ w_{pre}^M]^T \tag{10}$$

$$\mathbf{X}_{IQ} = \begin{bmatrix} x_{iq1,1} & x_{iq1,2} & \cdots & x_{iq1,M} \\ x_{iq2,1} & x_{iq2,2} & \cdots & x_{iq2,M} \\ \vdots & \vdots & \ddots & \vdots \\ x_{iqN,1} & x_{iqN,2} & \cdots & x_{iqN,M} \end{bmatrix} \tag{11}$$

$$\mathbf{y} = [y_1\ y_2\ ...\ y_N] \tag{12}$$

where $(.)^H$ is the Hermitian transpose. By applying the FFT operation to $\mathbf{y}$ we obtain the frequency-domain data $\tilde{\mathbf{y}} = [\tilde{y}_1\ \tilde{y}_2\ ...\ \tilde{y}_N]$.

The pre-FFT weights must be adjusted adaptively in defined intervals of received symbols. By comparing the received pilot symbols with their known values in the receiver, an error signal can be obtained. There are a total of P pilot symbols available. We define two $N \times 1$ vectors $\tilde{\mathbf{d}}_P$ and $\tilde{\mathbf{y}}_P$. The ith element of $\tilde{\mathbf{d}}_P$ is zero if the index i corresponds to a data subcarrier and is the known pilot value if index i corresponds to a pilot subcarrier. Similarly, the ith element of $\tilde{\mathbf{y}}_P$ is zero if the index i corresponds to a data subcarrier and is the received pilot value if index i corresponds to a pilot subcarrier. Therefore, the error signal in frequency-domain can be written as

$$\tilde{\mathbf{e}}_P = \tilde{\mathbf{d}}_P - \tilde{\mathbf{y}}_P \tag{13}$$

This error signal is converted to time-domain by an IFFT operation allowing to obtain $\mathbf{e} = [e(1)\ e(2)\ ...\ e(N)]^T$. The Pre-FFT weights are then updated using a *normalized least mean squared* (NLMS) algorithm, described by

$$w_{pre}^m(q) = w_{pre}^m(q-1) + \frac{\mu\, \mathbf{X}_{IQ}^{[m]\,T}(q)\mathbf{e}^*(q)}{a + \mathbf{X}_{IQ}^{[m]\,H}(q)\mathbf{X}_{IQ}^{[m]}(q)} \tag{14}$$

$$m = 1, 2, ..., M$$

where μ is the step-size, a is a small positive constant and $(.)^{[m]}$ denotes the m-th column of a matrix. $\mathbf{w}_{pre}(q)$ is applied to a defined interval of OFDM symbols, normally 3 or 4 symbols, and then updated.

2.2.2 *Post-FFT*

In Post-FFT beamforming the received time signal is first translated to frequency domain. Beamforming is then applied on each subcarrier. By defining $\tilde{x}_{iq(k,m)}$, the k-th subcarrier of the m-th antenna, we can write the received signal of the k-th subcarrier as

$$\tilde{y}(k) = \sum_{m=1}^{M} \tilde{w}^*_{(k,m)} \tilde{x}_{iq(k,m)} \quad 1 \le k \le N \tag{15}$$

where $\tilde{w}_{(k,m)}$ represents the weight associated with $\tilde{x}_{iq(k,m)}$, and all subcarriers have an associated weight. As only a few subcarriers are pilots, a group of subcarriers are clustered under one pilot symbol and the weight of that pilot symbol is applied to all the data subcarriers in the cluster. The error signal in the frequency-domain is found in the same way as in the Pre-FFT scheme. However, in the Post-FFT is used directly in frequency-domain to the weights update equation, to obtain

$$\tilde{\mathbf{W}}^{[m]}(q) = \tilde{\mathbf{W}}^{[m]}(q-1) + \frac{\mu\, \tilde{\mathbf{X}}_{IQ}^{[m]\,T}(q)\tilde{\mathbf{e}}_P^*(q)}{a + \tilde{\mathbf{X}}_{IQ}^{[m]\,H}(q)\tilde{\mathbf{X}}_{IQ}^{[m]}(q)} \tag{16}$$

$$m = 1, 2, ..., M$$

where $\tilde{\mathbf{W}}$ is now a matrix of size $N \times M$,

$$\tilde{\mathbf{W}} = \begin{bmatrix} \tilde{w}_{1,1} & \tilde{w}_{1,2} & \cdots & \tilde{w}_{1,M} \\ \tilde{w}_{2,1} & \tilde{w}_{2,2} & \cdots & \tilde{w}_{2,M} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{w}_{N,1} & \tilde{w}_{N,2} & \cdots & \tilde{w}_{N,M} \end{bmatrix} \tag{17}$$

3 PROPOSED ALGORITHM

Figure 1 illustrates the proposed Post-FFT beamforming with I/Q imbalance compensation algorithm. To take into account I/Q imbalance, we double the number of weights to apply them to the complex conjugate of the output of the FFTs operations. Accordingly, our received signal and weight matrices in frequency-domain, both with size $N \times 2M$, are given by

$$\tilde{\mathbf{X}}_C = [\tilde{\mathbf{X}}_{IQ}\ \tilde{\mathbf{X}}_{IQ}^{\#}] \tag{18}$$

$$\tilde{\mathbf{W}}_C = \begin{bmatrix} \tilde{w}_{1,1} & \tilde{w}_{1,2} & \cdots & \tilde{w}_{1,2M} \\ \tilde{w}_{2,1} & \tilde{w}_{2,2} & \cdots & \tilde{w}_{2,2M} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{w}_{N,1} & \tilde{w}_{N,2} & \cdots & \tilde{w}_{N,2M} \end{bmatrix} \tag{19}$$

where in Eq. (18) $(.)^{\#}$ denotes mirror complex conjugate of the column of a matrix, i.e.

$$\tilde{\mathbf{X}}_{IQ}^{\#} = \begin{bmatrix} \tilde{x}^*_{iq1,1} & \tilde{x}^*_{iq1,2} & \cdots & \tilde{x}^*_{iq1,M} \\ \tilde{x}^*_{iqN,1} & \tilde{x}^*_{iqN,2} & \cdots & \tilde{x}^*_{iqN,M} \\ \tilde{x}^*_{iqN-1,1} & \tilde{x}^*_{iqN-1,2} & \cdots & \tilde{x}^*_{iqN-1,M} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{x}^*_{iq2,1} & \tilde{x}^*_{iq2,2} & \cdots & \tilde{x}^*_{iq2,M} \end{bmatrix} \tag{20}$$

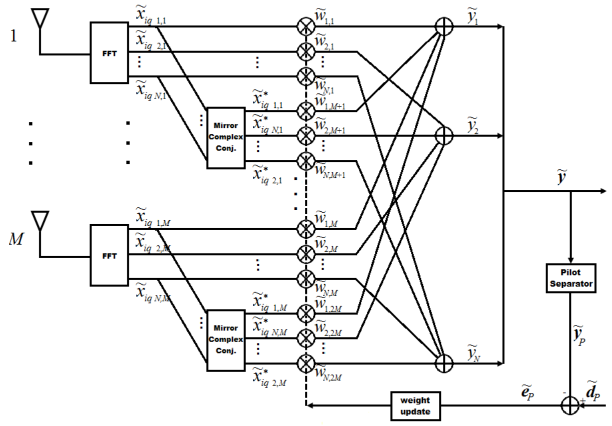

Figure 1: The proposed Post-FFT beamforming with I/Q imbalance compensation scheme.

Now to build the signal that will be demodulated we have,

$$\tilde{\mathbf{y}}_C = S(\tilde{\mathbf{W}}_C^* \circ \tilde{\mathbf{X}}_C) \tag{21}$$

where $\circ$ is the Hadamard product (i.e., the product element by element), and S is the operator for the sum of the elements in each row of the matrix. That operation results in a column vector of size $N \times 1$. Using that we can obtain the error signal vector, in a similar form than was described before in Eq. (13). In order to update the weights we use the following NLMS algorithm

$$\tilde{\mathbf{W}}_C^{[i]}(q) = \tilde{\mathbf{W}}_C^{[i]}(q-1) + \frac{\mu \tilde{\mathbf{X}}_C^{[i]\,T}(q)\tilde{\mathbf{e}}_P^*(q)}{a + \tilde{\mathbf{X}}_C^{[i]\,H}(q)\tilde{\mathbf{X}}_C^{[i]}(q)} \tag{22}$$

$$i = 1, 2, ..., 2M$$

As stated in the Pre-FFT and Post-FFT schemes, $\tilde{\mathbf{W}}_C(q)$ is applied to a defined interval of OFDM symbols and then updated.

4 EVALUATION AND SIMULATION RESULTS

The evaluation performed using simulations considers a comparison between the ideal case without I/Q imbalance, with I/Q imbalance and the known Post-FFT algorithm, and with I/Q imbalance and the proposed algorithm.

For all the results presented in this paper, we make the following assumptions: we assume 4 reception antennas; an OFDM system perfectly synchronized; a CP length of 32 and $N = 128$ subcarriers where 18 of them are pilots (distributed one per 6 data in the OFDM symbol), the central subcarrier in every cluster is taken as a pilot; a 16-QAM modulation scheme is employed; convolutional encoding with $R = 1/2$ is used; one desired source and

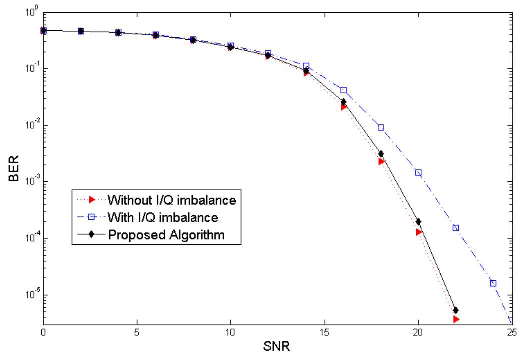

Figure 2: BER plots of the three cases: ideal case without I/Q imbalance and the known Post-FFT algorithm, with I/Q imbalance and the known Post-FFT algorithm, and with I/Q imbalance and the proposed algorithm

two interferents, all with the same power ($SIR = 0dB$), which were placed at 0, -60 and 75 degrees respectively; we assumed normalized multipath fading channels with 6 taps and power profile $P = [0 - 3 - 6 - 9 - 12 - 20]$ dB and delay profiles equal to $[0, 1, 2, 3, 4, 5]$; we assume 10% and 10 degrees of amplitude and phase imbalance respectively, and an actualization rate of weigths of 3, i.e.: one symbol with pilots, three symbols with only data, one symbol with pilots.

To elaborate the results presented, various realizations of the channels where made and then averaged in order to evaluate the performance of the algorithm in different situations. Perfect channel state information (CSI) is assumed at the receiver.

Figure 2 displays the BER curves of the three cases as a function of the input SNR; as it is seen, the BER curve of the proposed algorithm shows a much better performance that the system with the normal Post-FFT scheme, and is almost equal to the curve of the ideal case without I/Q imbalance.

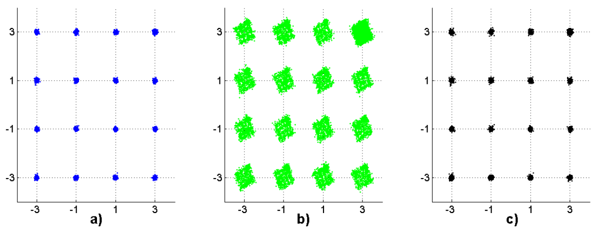

Figure 3: Constellation maps with $SNR = 40dB$ for the three cases: a) ideal case without I/Q imbalance and the known Post-FFT algorithm, b) with I/Q imbalance and the known Post-FFT algorithm, and c) with I/Q imbalance and the proposed algorithm.

In Fig. 3 the constellation maps for the three cases are shown, from them it's clear that the behavior of the results of the proposed algorithm are similar to the ideal case without I/Q imbalance. As presented in Fig. 4, there are no variations in the beamforming pattern between the ideal case and proposed algorithm presenting the nulls of it to the interfering signals, while the known Post-FFT scheme with I/Q imbalance presents a slightly variation.

5 CONCLUSIONS

In this paper, we propose a NLMS Post-FFT beamforming algorithm with the possibility to mitigate the I/Q imbalance produced by the down-conversion at the receiver side without losing its abilities to reject interfering signals. A reception scenario was presented with interference signals in a Rayleigh multipath fading channel with AWGN. Performance in terms of BER curves and constellation maps have been derived, providing that the proposed algorithm behaves in a very close form to the ideal case without I/Q imbalance.

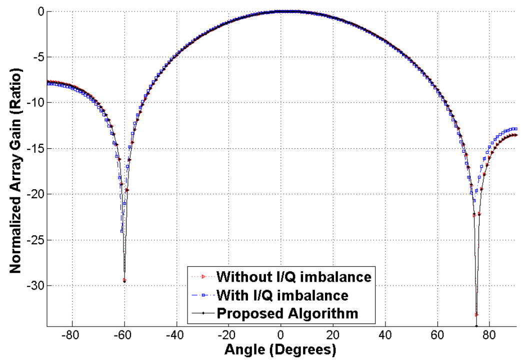

Figure 4: Beamforming patterns showing interference rejection of the three cases: ideal case without I/Q imbalance and the known Post-FFT algorithm, with I/Q imbalance and the known Post-FFT algorithm, and with I/Q imbalance and the proposed algorithm.

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